

A categorical approach to high-assurance science

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Science and map-making

Map-making: The earliest known scientific pursuit.

- Humans have been making maps for at least 8,000 years.
- They made maps of land and maps of the night sky.
- Different people made different observations.
- Comparing these observations and finding commonalities led to
 - increased reliability (peer review),
 - increased scope (division of labor),
 - increased cohesiveness of culture.

First view of the night sky



Second view of the night sky: saccade right

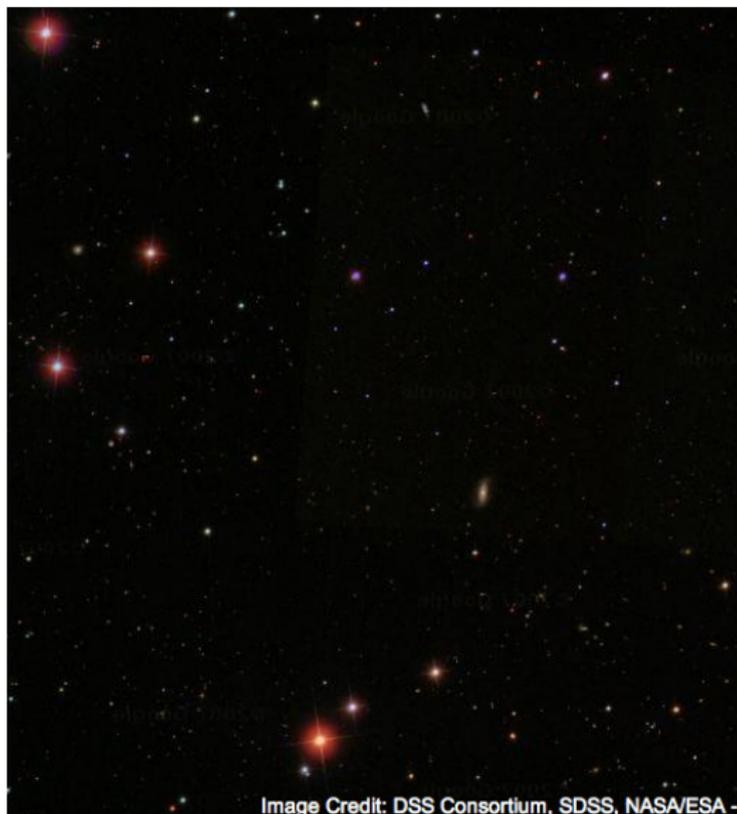


Image Credit: DSS Consortium, SDSS, NASA/ESA -



Third view of the night sky: saccade down-left



Image Credit: DSS Consortium, SDSS, NASA/ESA



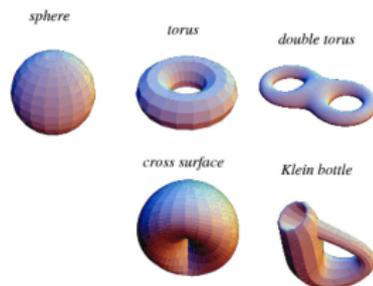
Three views put side by side



Three views put together: one big picture



Manifolds



(Source: Wolfram MathWorld)

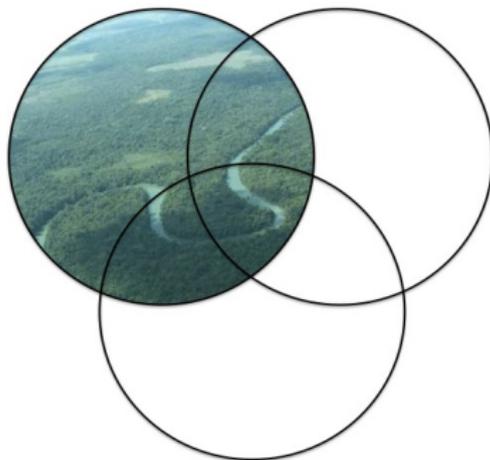
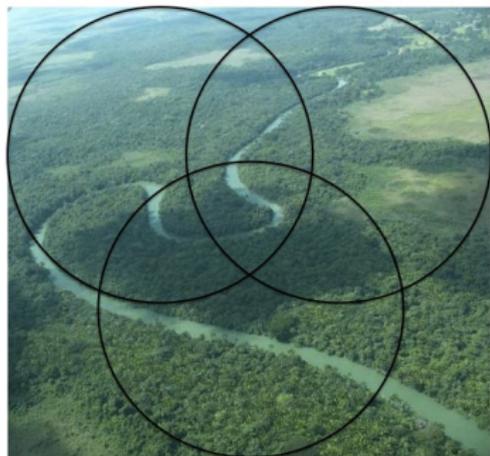
A manifold M is a mathematical shape (a topological space).

- It can be arbitrarily complex.
- It can have arbitrarily high dimension (e.g. $\dim(M) = 58$).

Two rules set manifolds apart:

- Locally, a manifold is completely understandable.
 - Each point has a neighborhood that's equivalent to \mathbb{R}^n .
- Each set of overlapping local pictures can be meaningfully compared.
 - Overlapping \mathbb{R}^n 's are compared by diffeomorphisms.

Overlapping views



Stitch together isomorphic but unequal overlaps (sheaves)



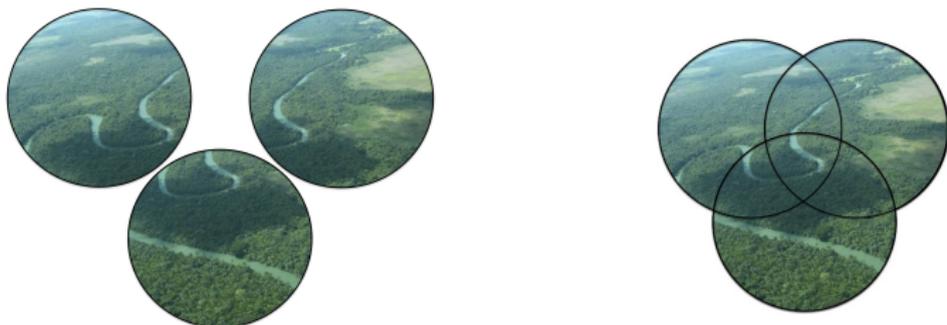
Different
angle



Different
color

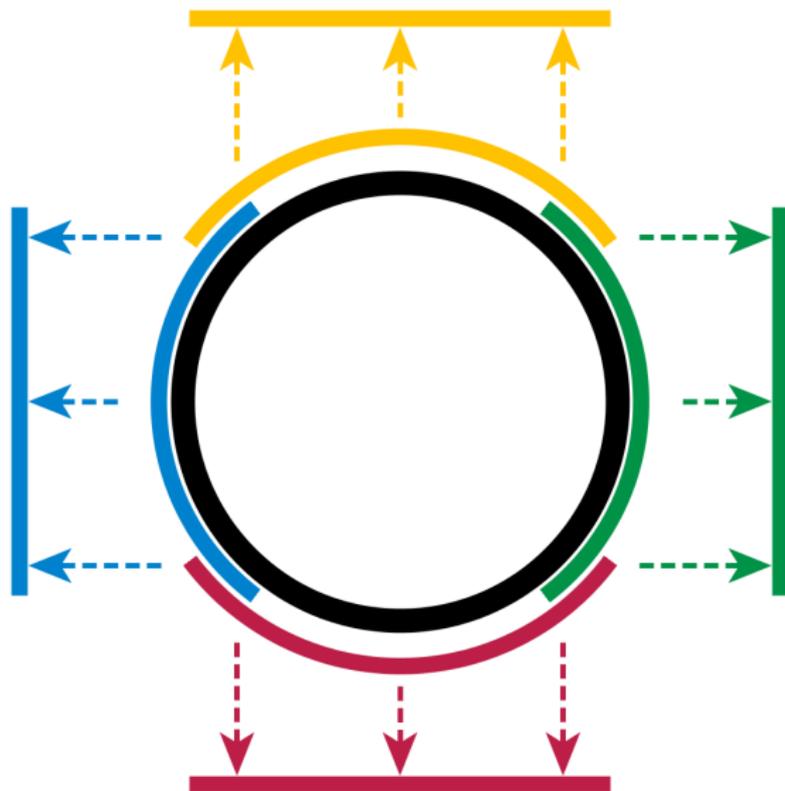


Linked local charts = Atlas



- Each local picture is called a *chart*.
- These charts are linked together by finding equivalent sub-charts.
- Together this system of linked charts forms an *atlas*.
- Crucial feature: controlled linkages
 - We have defined in advance the structure of each local picture.
 - We have defined in advance the structure of comparison on overlaps.

Atlas: Four overlapping views of the black circle



From [wikipedia.org](https://en.wikipedia.org).

Expanding on the manifolds idea

- Humans are information processing agents.
- We construct large-scale understanding out of simple local pictures.
 - Worldwide geographical information has been compiled from local data.
 - How are do we put together the mountains of local data into a complete picture?
 - Like a jigsaw puzzle: We look for agreement and fasten along it.
- This basic charts-to-atlas idea underlies the pursuit of knowledge.
- Can we use the charts-to-atlas formalism more broadly?

Individuals as explorers

Imagine that each person is an explorer of his or her world.

- This world is more than spatial, more than visual:
- It includes every kind of information.
- Anything that can be classified, understood, or made sense of.
 - Physics, chemistry, mathematics,
 - fighting, war, logistics,
 - courtesy, relationship, cultures,
 - history, linguistics, psychology.
- Humans have explored a vast informational territory.

Where we are now

Each explorer (person) has made sense of a swath of the world.

- Individual entities (scientists, businesses) often have mature understandings.
- How is this knowledge passed along?
- Information is shared in a weak way:
 - by word of mouth,
 - by imitation,
 - by prose text.
- We need a more robust, rigorous way to share information.

Analogy to software

- To make a working program, one can cobble something together.
- Weakly-typed languages (Perl) are useful for quickly producing individual scripts.
- Strongly-typed languages (Haskell, ML) are much more robust and safe.
 - High-assurance software.
 - Haskell is used in NSA projects (Galois), pharmaceutical companies (Amgen).
 - More scalable, easier to build on, longer life.
- Difference: what is passed
 - strongly-typed languages pass values having ready-made interpretation.
 - weakly-typed languages pass values requiring dynamic interpretation.

Creating high-assurance science

- Individual scientists perform experiments and determine values.
 - For example, the Young's modulus of a material.
 - The throughput of a transportation network.
- How are these results passed from researcher to researcher?
 - In paper publications, as prose text.
 - In talks, as spoken English.
 - By imitation of lab-mates, advisors.
 - These all require human judgment, educated guessing.
- We can do better: strongly typed ideas.

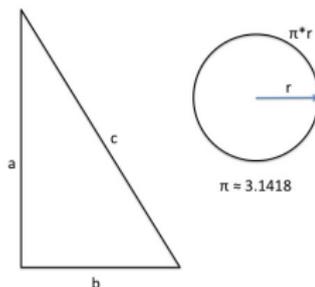
Firm foundations: mathematics and reality

Good science is grounded in two worlds:

- the real world of observation and experiment, and
- the conceptual world of rigorous mathematics.

Example: Geometry and map-making

- Geometry was contemporary with early map-making.
 - Pythagorean theorem known 4,000 years ago.
 - π known to be about 3.1418 by Archimedes in 250 BC.
- Mathematics served as a check for map-making:
 - The data of observation had to conform with geometric principles.
 - Non-conforming maps may have served temporarily as a heuristic, but they would fall apart under stress or scrutiny.



Pythagorean theorem: $a^2+b^2=c^2$

Good science includes good communication of science

- Today, scientific studies are already firmly grounded in mathematics.
- But science is much more than individual studies.
- It is a network of scientists learning from each other.
- The *communication* of science must be made formal and rigorous.

What is needed from mathematics?

Mathematics can provide:

- A language in which to carefully record all sorts of information.
- A computational toolset with which to manipulate the information.
- A mathematical basis for an atlas of scientific ideas:
 - Each scientist is an explorer, mapping out some territory.
 - We want to rigorously connect these charts into an atlas of human knowledge.

Without mathematics, this would be just a pie-in-the-sky idea.

- The goal of today's talk is to suggest an approach to formalizing it.
- Essential ingredient: connecting databases and category theory.
- This allows us to create an atlas of databases.

Outline:

We want to connect information and mathematics.

- 1 Discuss what information is, and how we work with it currently.
- 2 Discuss what category theory is.
- 3 Show the essential similarity between these subjects.
- 4 Discuss linkages between disparate viewpoints.
- 5 Review.

What is information?

- There is plenty of information being produced and used.
- But it is hard to say exactly what information *is*.
- Some sources of information:
 - Dictionaries.
 - Engineer's schematic diagrams.
 - Architect's floor plans.
 - Databases.
- So... what is it?

In formation



What is information?

- Controlled formation!
 - Controlling formation is the same as enforcing order, dispelling chaos.
 - It obviates guessing.
 - It promotes effective reasoning.
 - Information is always in formation.
- Our sources of information:
 - Dictionaries.
 - Engineer's schematic diagrams.
 - Architect's floor plans.
 - Databases.
- Easiest to mathematize: databases.
 - Databases bridge the divide between theory and practice.
 - They include both conceptual layout and on-the-ground facts.

What is a database?

- A database consists of a schema and conforming data.
- Database schema (conceptual layout).
 - A collection of tables.
 - Each table has many columns.
 - The columns refer us from one table to another.
- Database instance (on-the-ground facts).
 - A database instance is a collection of data.
 - Each table is filled with rows of data.
 - All the data is in accordance with the schema.

Example database instance

A family of linked tables:

| dog | | | |
|------|--------|-------|---------------|
| ID | name | owner | address |
| D101 | Wally | P34 | 15 Ash St. |
| D102 | Fido | P46 | 201 5th Ave. |
| D104 | Buster | P17 | 27 Spring Ln. |

| person | | |
|--------|----------|-----------------|
| ID | lastName | address |
| P17 | Jones | 27 Spring Ln. |
| P19 | Smith | 201 Gladys Ave. |
| P34 | Smith | 15 Ash St. |
| P46 | D'Angelo | 201 5th Ave. |

| dogName |
|-----------|
| ID |
| Barkie |
| Buster |
| Fido |
| Puppers |
| Rosie |
| Samson |
| Wally |

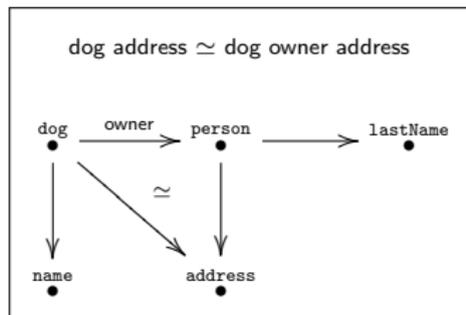
| address |
|-----------------|
| ID |
| 15 Ash St. |
| 27 Spring Ln. |
| 201 5th Ave. |
| 201 Gladys Ave. |

| lastName |
|-----------|
| ID |
| Bennet |
| D'Angelo |
| Jimenez |
| Jones |
| Moran |
| Smith |
| Vickers |

Database schemas enforce order

- A family of tables is organized by a data architect.
- This organizational structure is called a *schema*.
- A schema specifies precisely:
 - The set of tables and their columns.
 - How the tables interrelate.

A database and its schema



| dog | | | |
|------|--------|-------|---------------|
| ID | name | owner | address |
| D101 | Wally | P34 | 15 Ash St. |
| D102 | Fido | P46 | 201 5th Ave. |
| D104 | Buster | P17 | 27 Spring Ln. |

| person | | |
|--------|----------|-----------------|
| ID | lastName | address |
| P17 | Jones | 27 Spring Ln. |
| P19 | Smith | 201 Gladys Ave. |
| P34 | Smith | 15 Ash St. |
| P46 | D'Angelo | 201 5th Ave. |

| name |
|--------|
| ID |
| Buster |
| ⋮ |
| ⋮ |

| address |
|------------|
| ID |
| 15 Ash St. |
| ⋮ |
| ⋮ |

| lastName |
|----------|
| ID |
| D'Angelo |
| ⋮ |
| ⋮ |

Goal: a mathematical foundation for databases

- The world's information is stored in databases.
- I wanted to find a mathematical basis for databases which:
 - Completely describes schemas, instances, and the relationship between them.
 - Formalizes all typical database operations and querying.
 - Simplifies schema evolution, data migration, and database merging.
 - Links with other information paradigms (RDF and programming languages).
 - Offers new insights and tools.
- The simpler, the better.

What is category theory?

- Since its invention in the early 1940s, category theory has revolutionized math.
- It's like set theory and logic, except less floppy, more principles-based.
- It was invented to build bridges between disparate branches of math by distilling the essence of mathematical structure.
- Original use: connecting topology and algebra.
 - The essence of each was formulated as a category.
 - Rigorous mappings (functors) were established, connected these two categories.
 - These mappings were used to import theorems from algebra as new theorems in topology.

Category theory: branching out

- Category theory naturally fosters connections between disparate fields.
- It has branched out of math and into physics, linguistics, materials science, and biology.
- It has had much success in computer science.
 - Specifically important in the theory of programming languages.
 - The category-theoretic concept of *monads* has vastly extended the reach of functional programming.
- Success comes from simplicity.

Definition of a category I: Constituents

A category \mathcal{C} consists of the following constituents:

- ① A set $\mathbf{Ob}(\mathcal{C})$, called *the set of objects of \mathcal{C}* .
 - I'll denote each object $x \in \mathbf{Ob}(\mathcal{C})$ by \bullet^x .
- ② A set $\mathbf{Arr}(\mathcal{C})$, called *the set of arrows of \mathcal{C}* , and two functions

$$\mathit{src}, \mathit{tgt}: \mathbf{Arr}(\mathcal{C}) \rightarrow \mathbf{Ob}(\mathcal{C}),$$

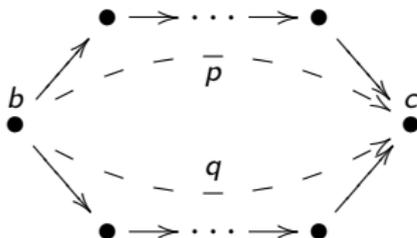
assigning to each arrow its *source* and its *target* object, respectively.

- An arrow $f \in \mathbf{Arr}(\mathcal{C})$ is often written $\bullet^x \xrightarrow{f} \bullet^y$, where $x = \mathit{src}(f), y = \mathit{tgt}(f)$.
 - We define a *path in \mathcal{C}* to be a finite “head-to-tail” sequence of arrows in \mathcal{C} , e.g. $\bullet^x \xrightarrow{f} \bullet^y \xrightarrow{g} \bullet^z$.
 - Paths can have length n for any $n \in \mathbb{N}$, including $n = 0$ and $n = 1$.
- ③ An notion of equivalence for paths, denoted \simeq .

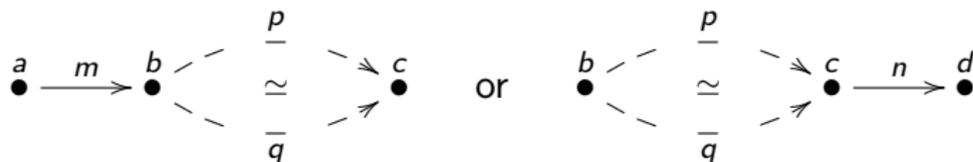
Definition of a category II: Rules

These constituents must satisfy the following requirements:

- 1 If $p \simeq q$ are equivalent paths then the sources agree: $\text{src}(p) = \text{src}(q)$.
- 2 If $p \simeq q$ are equivalent paths then the targets agree: $\text{tgt}(p) = \text{tgt}(q)$.
- 3 Suppose we have two paths (of any lengths) $b \rightarrow c$:



If $p \simeq q$ then for any extensions



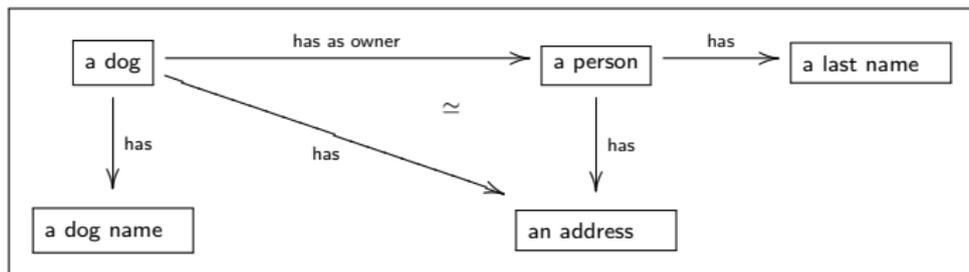
$$m; p \simeq m; q$$

and

$$p; n \simeq q; n$$

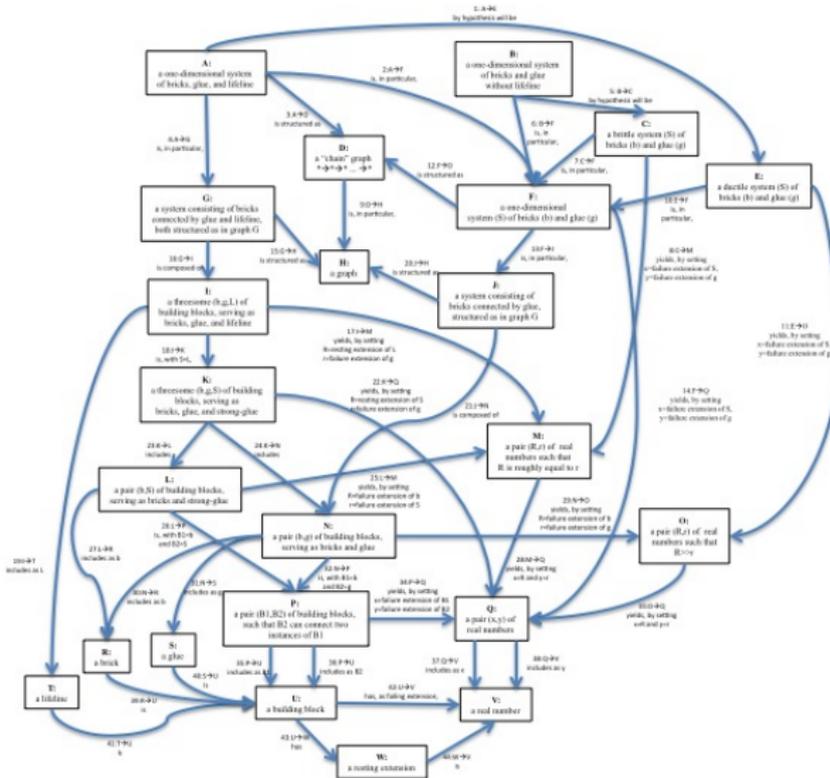
Ologs connect people, databases, and categories

- It turns out that categories and database schemas have the same structure!
- I call the connection between them ologs.



- An olog is both a database schema and a category.

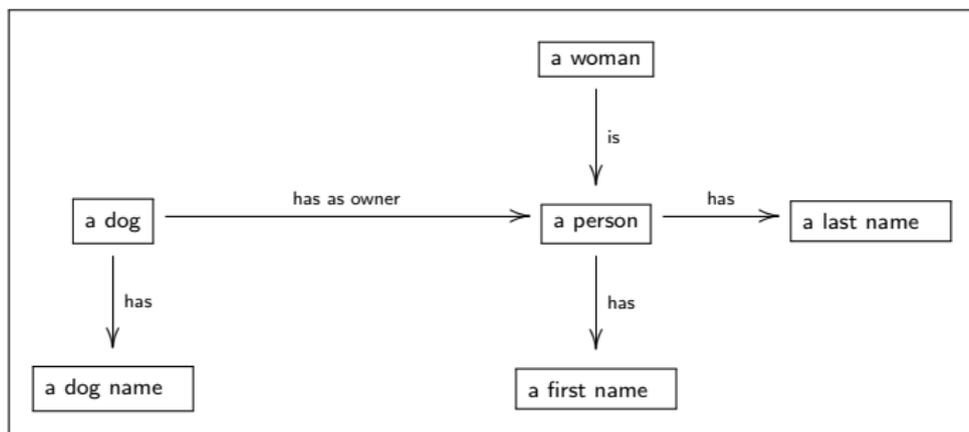
Example: an olog describing hierarchical protein materials



What is an olog?

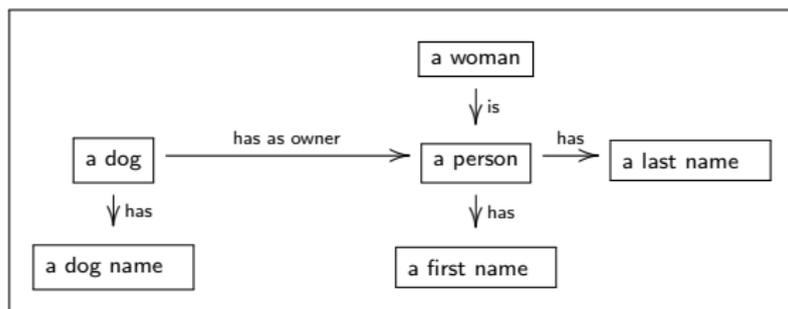
- An olog is a conceptual description of a subject.
- Olog stands for “ontology log”
 - Ontology is the study of what something *is*.
 - “Log” because the study is never complete—always expanding.
- Components of an olog:
 - Labeled boxes,
 - Labeled arrows,
 - Path equivalences.

Ologs are database schemas 1: an example olog



- Boxes are tables
- Arrows are columns.
 - We can predict how many columns the **a dog** table will have.

Ologs are database schemas 2: database

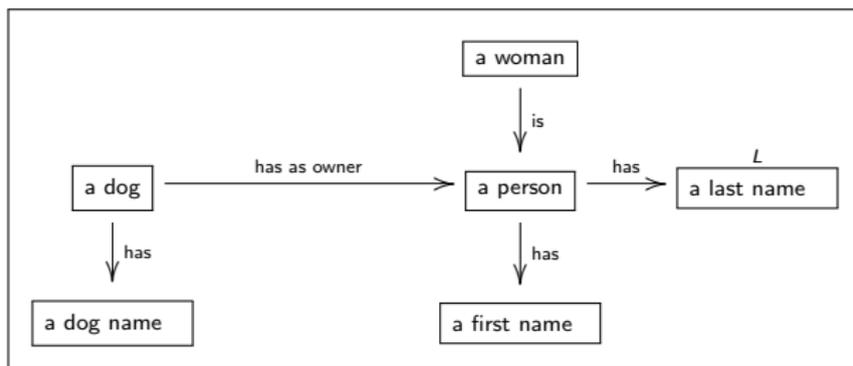


| a woman | |
|---------|-------------|
| ID | is a person |
| W17 | P17 |
| W34 | P34 |
| W38 | P38 |
| W51 | P51 |

| a dog | | |
|-------|--------------------------|----------------|
| ID | has as owner a person | has a dog name |
| D101 | P34 | Wally |
| D102 | P46 | Fido |
| D103 | P34 | Samson |
| D104 | P17 | Buster |
| D106 | P19 | Rosie |

| a person | | |
|----------|------------------|-----------------|
| ID | has a first name | has a last name |
| P17 | Alice | Jones |
| P19 | Bob | Smith |
| P34 | Barbara | Smith |
| P38 | Sandra | Moran |
| P46 | Jeremy | D'Angelo |
| P51 | Luisa | Jimenez |

Leaf tables

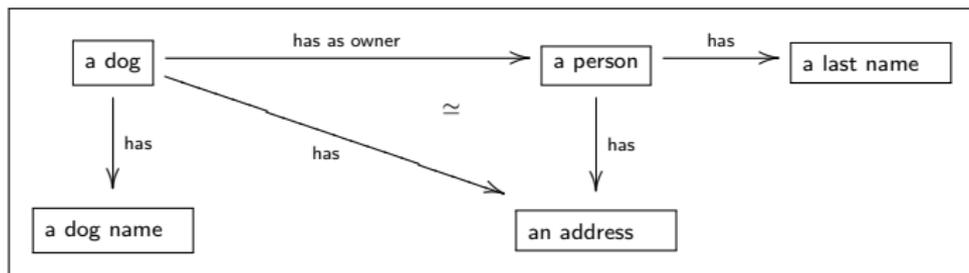


| a dog name | |
|------------|--|
| ID | |
| Barkie | |
| Buster | |
| Fido | |
| Puppers | |
| Rosie | |
| Samson | |
| Wally | |

| a first name | |
|--------------|--|
| ID | |
| Alice | |
| Bob | |
| Barbara | |
| Carl | |
| Jeremy | |
| Luisa | |
| Sandra | |
| Thomas | |

| a last name | |
|-------------|--|
| ID | |
| Bennet | |
| D'Angelo | |
| Jimenez | |
| Jones | |
| Moran | |
| Smith | |
| Vickers | |

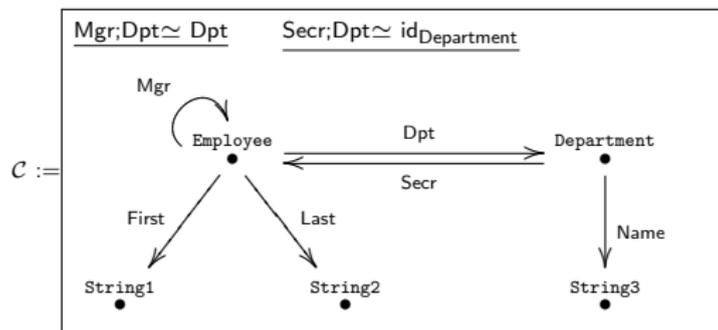
Equivalent paths require equivalent data



| dog | | | |
|------|--------|-------|---------------|
| ID | name | owner | address |
| D101 | Wally | P34 | 15 Ash St. |
| D102 | Fido | P46 | 201 5th Ave. |
| D104 | Buster | P17 | 27 Spring Ln. |

| person | | |
|--------|----------|-----------------|
| ID | lastName | address |
| P17 | Jones | 27 Spring Ln. |
| P19 | Smith | 201 Gladys Ave. |
| P34 | Smith | 15 Ash St. |
| P46 | D'Angelo | 201 5th Ave. |

Another example of path equivalences



| Employee | | | | |
|----------|----------|---------|-----|-----|
| Id | First | Last | Mgr | Dpt |
| 101 | David | Hilbert | 103 | q10 |
| 102 | Bertrand | Russell | 102 | x02 |
| 103 | Alan | Turing | 103 | q10 |

| Department | | |
|------------|------------|------|
| Id | Name | Secr |
| q10 | Sales | 101 |
| x02 | Production | 102 |

| String |
|--------|
| Id |
| a |
| b |
| ⋮ |
| z |
| aa |
| ab |
| ⋮ |
| ⋮ |

Ologs bridge the divide

- Each olog is authored by an individual or group, about a subject.
 - The olog idea can be understood by ordinary people.
 - No database theory or category theory necessary.
- Ologs are both databases and categories, in disguise.
 - Ologs are database schemas; we can fill them with relevant data.
 - Ologs are categories; mathematics can be brought to bear.
- I will use the following words interchangeably:
 - Olog,
 - Database schema,
 - Category.

Relating different families of tables

- An olog is the mathematical structure underlying a database.
 - Each database is a specific layout for a whole family of tables.
- We want to link different ologs together.
- Example 1: Banks
 - Each bank has its own database schema.
 - No two banks structure their tables in exactly the same way.
 - The Federal Reserve wants to understand the whole picture.
- Example 2: Formal network of science.
- How can mathematics help?

Linking databases together

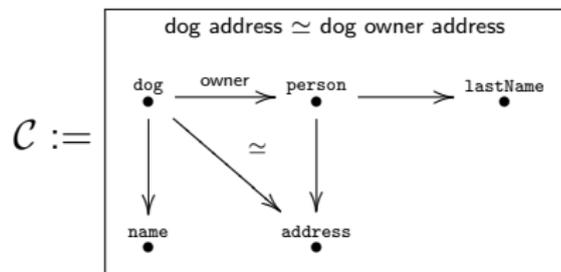
- Forming a coherent whole.
 - Different scientists or different banks may structure their data differently.
 - If they are studying the same subject, links should exist.
 - We want to stitch differently-structured schemas together.
 - Connecting different schemas is the same as connecting different categories.
- Category theory was designed specifically for this.
- Next we will discuss the links between categories, called *functors*.

Functors: mappings between categories

- One way to think of a category is as a directed graph, where certain paths have been declared equivalent.
- A functor is a graph-mapping that is required to respect equivalence of paths.
- **Definition:** A functor $F: \mathcal{C} \rightarrow \mathcal{D}$ consists of
 - a function $\mathbf{Ob}(\mathcal{C}) \rightarrow \mathbf{Ob}(\mathcal{D})$ and
 - a function $\mathbf{Arr}(\mathcal{C}) \rightarrow \mathbf{Path}(\mathcal{D})$,such that F
 - respects sources and targets,
 - respects equivalences of paths.

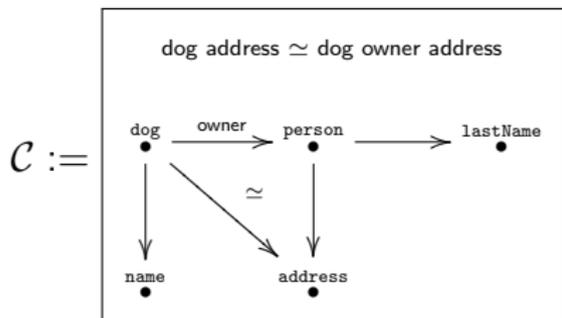
Backing up: a database instance is a functor!

- A database schema (layout of tables) is simply a category \mathcal{C} .



- There is a category **Set** of sets and functions.
- A functor $I: \mathcal{C} \rightarrow \mathbf{Set}$ assigns:
 - to each object $c \in \mathbf{Ob}(\mathcal{C})$ a set $I(c)$,
 - to each arrow $h: c \rightarrow d$ in \mathcal{C} a function $I(h): I(c) \rightarrow I(d)$,
 - such that all path equivalences are respected.
- In other words, a functor $I: \mathcal{C} \rightarrow \mathbf{Set}$ is a database instance on \mathcal{C} ; i.e. it is a way to fill \mathcal{C} with compatible data.

Example



We can represent a functor

$$I: \mathcal{C} \rightarrow \mathbf{Set}$$

as follows:

| dog | | | |
|------|--------|-------|---------------|
| ID | name | owner | address |
| D101 | Wally | P34 | 15 Ash St. |
| D102 | Fido | P46 | 201 5th Ave. |
| D104 | Buster | P17 | 27 Spring Ln. |

| person | | |
|--------|----------|-----------------|
| ID | lastName | address |
| P17 | Jones | 27 Spring Ln. |
| P19 | Smith | 201 Gladys Ave. |
| P34 | Smith | 15 Ash St. |
| P46 | D'Angelo | 201 5th Ave. |

| name |
|--------|
| ID |
| Buster |
| ⋮ |
| ⋮ |

| address |
|------------|
| ID |
| 15 Ash St. |
| ⋮ |
| ⋮ |

| lastName |
|----------|
| ID |
| D'Angelo |
| ⋮ |
| ⋮ |

Changes in schema

- Suppose in our modeling of a given subject, we evolve from schema \mathcal{C} to schema \mathcal{D} .
- We should find a functorial connection between them.
- Over time we may have something like

$$\mathcal{C} = \mathcal{C}_0 \xrightarrow{F_0} \mathcal{C}_1 \xrightarrow{F_1} \dots \xrightarrow{F_n} \mathcal{C}_n = \mathcal{D}$$

- We want to be able to migrate data from \mathcal{C} to \mathcal{D} and vice versa.
- We want to be able to migrate queries against \mathcal{C} to queries against \mathcal{D} and vice versa.
- And we want this all to work as expected.

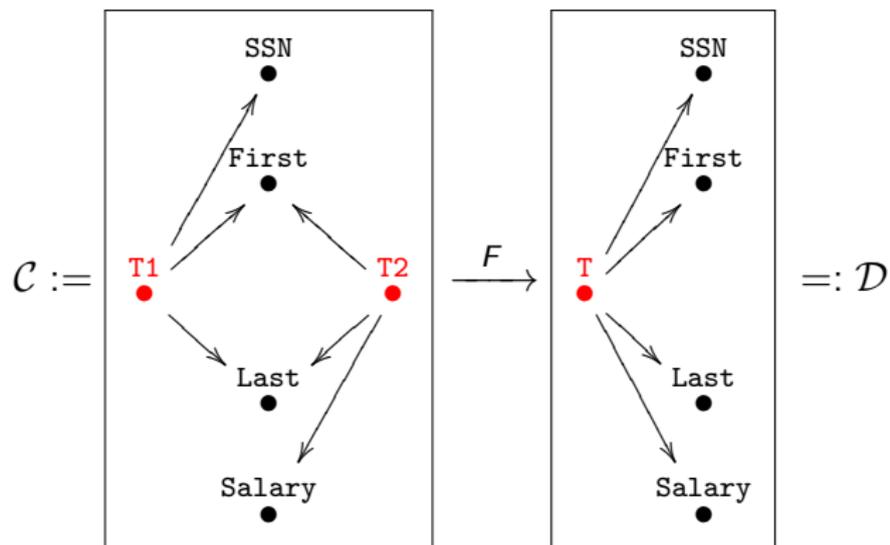
Functorial data migration for CT experts

- For any schema (category) \mathcal{C} , we have the category $\mathcal{C}\text{-Set}$ of set-valued functors $I: \mathcal{C} \rightarrow \mathbf{Set}$ and natural transformations. These are the instances of the database.
- A functor $F: \mathcal{C} \rightarrow \mathcal{D}$ serves as a translation between schemas.
- Composition with F induces a functor $\Delta_F: \mathcal{D}\text{-Set} \rightarrow \mathcal{C}\text{-Set}$,

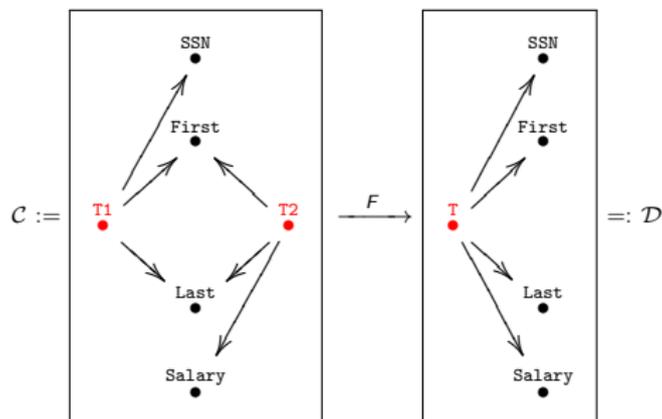
$$\mathcal{C} \xrightarrow{F} \mathcal{D} \xrightarrow{I} \mathbf{Set}.$$

- The functor Δ_F migrates data from \mathcal{D} back to \mathcal{C} .
- It has two adjoints $\Sigma_F: \mathcal{C}\text{-Set} \rightarrow \mathcal{D}\text{-Set}$ and $\Pi_F: \mathcal{C}\text{-Set} \rightarrow \mathcal{D}\text{-Set}$.

Uses of functorial data migration 1: Translation F



Uses of functorial data migration 2: Projection via Δ_F



$J: D \rightarrow \mathbf{Set}$:

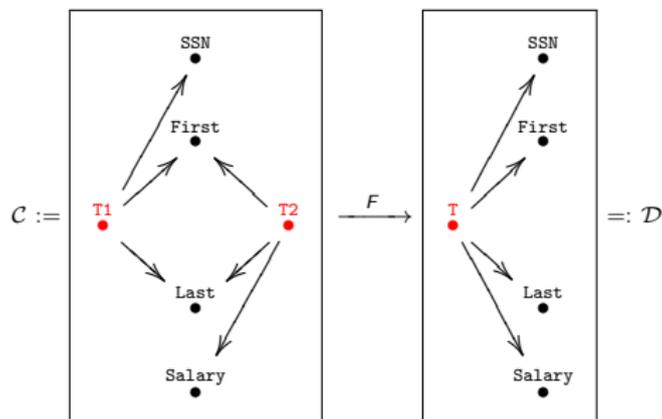
| T | | | | |
|-------|---------|-------|-------|--------|
| ID | SSN | First | Last | Salary |
| XF667 | 115-234 | Bob | Smith | \$250 |
| XF891 | 122-988 | Sue | Smith | \$300 |
| XF221 | 198-877 | Alice | Jones | \$100 |

$\Delta_F(J): C \rightarrow \mathbf{Set}$:

| T1 | | | |
|---------|---------|-------|-------|
| ID | SSN | First | Last |
| XF667T1 | 115-234 | Bob | Smith |
| XF891T1 | 122-988 | Sue | Smith |
| XF221T1 | 198-877 | Alice | Jones |

| T2 | | | |
|---------|-------|-------|--------|
| ID | First | Last | Salary |
| XF667T2 | Bob | Smith | \$250 |
| XF891T2 | Sue | Smith | \$300 |
| XF221T2 | Alice | Jones | \$100 |

Uses of functorial data migration 3: Joins via Π_F



$I: C \rightarrow \text{Set}$:

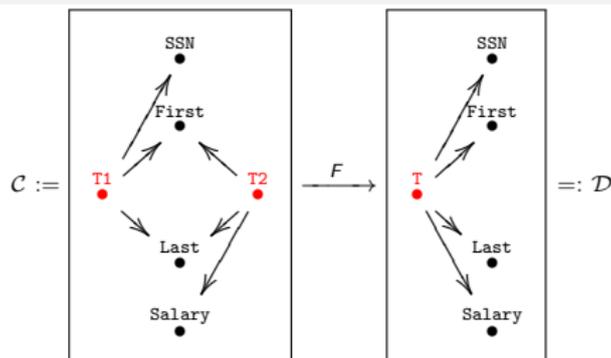
| T1 | | | |
|--------|---------|-------|-------|
| ID | SSN | First | Last |
| T1-001 | 115-234 | Bob | Smith |
| T1-002 | 122-988 | Sue | Smith |
| T1-003 | 198-877 | Alice | Jones |

| T2 | | | |
|---------|-------|--------|--------|
| ID | First | Last | Salary |
| T2-A101 | Alice | Jones | \$100 |
| T2-A102 | Sam | Miller | \$150 |
| T2-A104 | Sue | Smith | \$300 |
| T2-A110 | Carl | Pratt | \$200 |

$\Pi_F(I): D \rightarrow \text{Set}$:

| T | | | | |
|---------------|---------|-------|-------|--------|
| ID | SSN | First | Last | Salary |
| T1-002T2-A104 | 122-988 | Sue | Smith | \$300 |
| T1-003T2-A101 | 198-877 | Alice | Jones | \$100 |

Uses of functorial data migration 4: Unions via Σ_F



$I: C \rightarrow \mathbf{Set}$:

| T1 | | | |
|--------|---------|-------|-------|
| ID | SSN | First | Last |
| T1-001 | 115-234 | Bob | Smith |
| T1-002 | 122-988 | Sue | Smith |
| T1-003 | 198-877 | Alice | Jones |

| T2 | | | |
|---------|-------|--------|--------|
| ID | First | Last | Salary |
| T2-A101 | Alice | Jones | \$100 |
| T2-A102 | Sam | Miller | \$150 |
| T2-A104 | Sue | Smith | \$300 |
| T2-A110 | Carl | Pratt | \$200 |

$\Sigma_F(I): D \rightarrow \mathbf{Set}$:

| T | | | | |
|---------|-------------|-------|--------|---------------|
| ID | SSN | First | Last | Salary |
| T1-001 | 115-234 | Bob | Smith | T1-001.Salary |
| T1-002 | 122-988 | Sue | Smith | T1-002.Salary |
| T1-003 | 198-877 | Alice | Jones | T1-003.Salary |
| T2-A101 | T2-A101.SSN | Alice | Jones | \$100 |
| T2-A102 | T2-A102.SSN | Sam | Miller | \$150 |
| T2-A104 | T2-A104.SSN | Sue | Smith | \$300 |
| T2-A110 | T2-A110.SSN | Carl | Pratt | \$200 |

Category theory provides a foundation for information

- We can formulate an understanding of any topic using an olog.
- The olog can then be filled with conforming data.
 - Categories capture the structure of databases completely.
 - The olog structure points out variables that are critical to our understanding.
 - All in an intuitive yet rigorous way.
- All typical manipulations of the data are grounded in pure math.
 - Simple data shuffling (projections, unions, joins, warehousing, etc.)
 - Aggregations (sums, counts, averages).
 - Curve fitting, parameter estimation, classifying results.
 - Schema evolution, data migration, merging.

Category theory has 70 years worth of useful theorems

- There is perfect correspondence between database and categories.
- Theorems about categories are theorems about databases.
- We can thus *prove* things about how information works.

Ask me later about:

There wasn't space to fit in the following neat examples:

- Connection to RDF and semi-structured data via the Grothendieck construction.
 - SPARQL graph-pattern queries become topological lifting problems.
- A normal form for queries using a purely categorical theorem.
- Aggregation functions and hierarchical categories.
- Probabilistic ologs with Bayesian updating.
- Moggi-style use of monads in categorical databases.

Advantages of a mathematical foundation

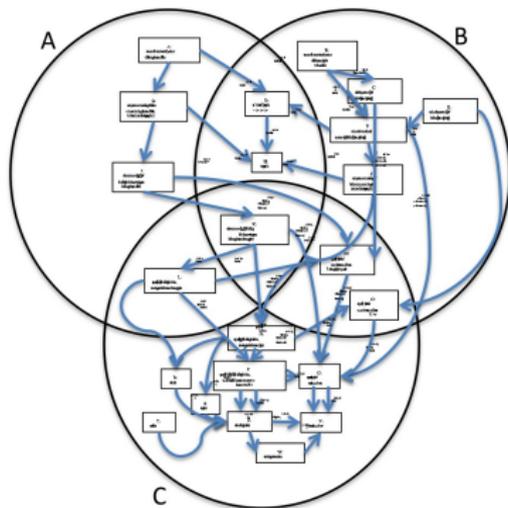
- A unified language for information science. (Interoperable)
- Easily adaptable to different platforms. (Portable)
- Database operations are founded and certifiable. (Rigorous)
 - We can prove things about the results of a query or data transformation before performing it.
 - Mathematics does not fail under pressure.
- The underlying mathematics is the same at all size scales. (Scalable)

Certified science

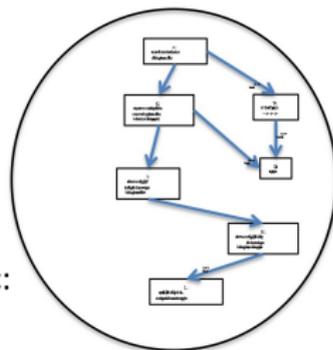
Envision this process:

- A scientific topic is formulated as an olog.
- A proof assistant (Coq) is used to transform the mathematical toolset described above into a certified program.
 - This program collects the data and performs the manipulations.
 - Parameter estimation, curve fitting, statistical diagnoses are all provable.
 - All scientific claims are proven, and others can investigate the evidence at any scale.
 - Information is more readily shared and processed.
 - At each step, a mathematical proof of correctness (certificate) is produced by Coq.
- Different topics can be fused together to create a robust network of human understanding.

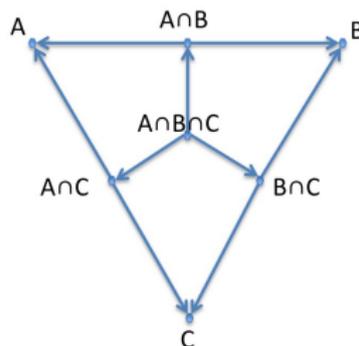
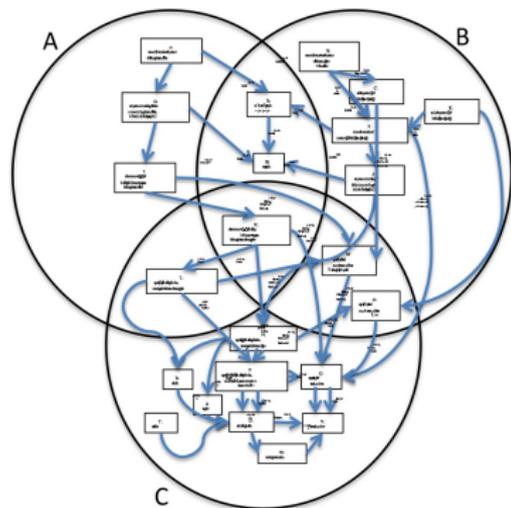
Network of scientists 1: overlapping understanding



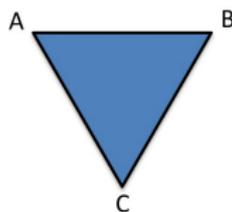
Scientist A's research topic:



Network of scientists 2: encoding interaction groups

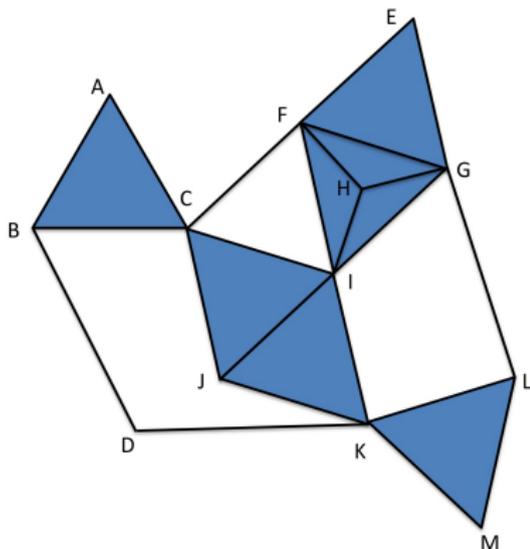


Abstraction:



Network of scientists 3: simplicial complex

A network of ologs, a network of scientific understanding.



This whole network can be queried, with provenance plainly evident.

A firm foundation for communication

We should study the communication process.

- Ground in the real world of observation and experiment.
 - How is information stored, processed, transferred currently?
 - This has not caught on as a mathematical pursuit.
- Check using the rigor of mathematics.
 - Find tried-and-true mathematical structures on which to base the study.
 - Practitioners are generally uninterested in this.
- A healthy combination will have profound affect.
 - Symbiosis between math and physics benefited both fields.
 - Could such a relationship exist between mathematics and information?

My growing network

- Professors in other fields at MIT
 - Markus Buehler, CEE
 - Adam Chlipala, CSAIL
- Mathematics postdocs and professors
 - Scott Morrison (UC Berkeley)
 - Nat Stapleton (MIT)
 - Mathieu Anel (UQAM)
 - David Gepner (Universität Regensburg)
 - Steve Awodey (CMU)
 - Jack Morava (JHU)
- Industry
 - Dave Balaban (VP at Amgen)
 - Peter Gates (Johnson and Johnson)
 - Rich Haney (GlaxoSmithKline)
 - Carlo Curino (Yahoo! Research)
 - Allen Brown (Microsoft Research)

Summary of the talk

- We need to improve our ability to communicate rigorously about complex subjects.
 - Transferring knowledge from one group to another is difficult.
 - It cannot be left to human guessing and ad-hoc interpretation.
 - We need to have available a high-assurance framework for communication.
- Ologs and category theory provide such a framework.
- If broadly adopted, this could have profound impacts on science.
 - A formal strategy for stitching local databases into an atlas of science.
 - Unify each field in vocabulary and agenda.
 - Improve data acquisition strategies.
 - Allow multi-lab data analysis.
- Scientific communication will surely benefit from an infusion of mathematics.

Thank you

Thanks for inviting me to speak!