OPERADS, AND THEIR ALGEBRAS, FOR BUILDING NEW PROCESSORS FROM OLD

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ABSTRACT. In this talk, I'll write down a very simple (colored) operad O that models information processing in a network, or material flow through a system. Basically, the morphisms of O look like boxes wired together inside a box; the outer box as a process is constituted by the wiring pattern of its constituents. I'll discuss an O-algebra of state machines, as well as perhaps some monadic extensions. I'll also discuss a connection between the algebras on a wiring diagram operad and traced monoidal categories. This is joint work with Dylan Rupel and Nat Stapleton.

(Thank Eugene Lerman)

I. Introduction
   A. Why I care.
      1. CT for information and communication
      2. Databases
      3. Circuits and Petri nets
      4. Brains
         a. My work on undirected WDs.
   B. Work with Nat Stapleton
      1. Relating databases and programs
         a. Developed the operad of directed wiring diagrams
         b. Formalized what a wiring diagram is
            (1) Wiring proposal p: Lins+Gouts → Gins+Louts
            (2) Underlying pre-orders should be partial.
      2. Unfinished work to take up later.
         a. Never wrote it up.
         b. Didn't answer all questions.
            (1) Example: do these model Cartesian categories somehow?
   C. Work with Dylan Rupel
      1. Brains & minds
         a. Brief foray into neural networks.
         b. Very disappointing (couldn't get identity or greater-than).
      2. We need a language for these ideas.
      3. Picked up where Nat & I left off, and added feedback.
      4. Everything today is joint work with Dylan.
   D. Outline of talk:
      1. The operad / SMC of wiring diagrams
      2. The algebra of propagators
      3. Applications of this kind of setup
      4. Relationship to string diagrams and traced monoidal categories

II. The operad / SMC of wiring diagrams
   A. Terminology and notation: operads
      1. "Operad" = symmetric colored operad

2. Relationship to SMCs

B. The operad **W**.

1. **TFS** — typed finite sets.
2. $\text{Ob}(\mathbf{W}) = \text{Ob}(\mathbf{TFS} \times \mathbf{TFS})$
3. $\text{Hom}_{\mathbf{W}}([A, B], [A', B'])$

a. First pass

b. After streamlining

$$\texttt{int}(\phi) \subseteq \texttt{inp}(X)$$

$$\phi^{in} \colon \texttt{inp}(X) \longrightarrow \texttt{int}(\phi) + \texttt{inp}(Y)$$

$$\phi^{out} \colon \texttt{int}(\phi) + \texttt{out}(Y) \longrightarrow \texttt{out}(X)$$

$$\text{inclusion: } \texttt{int}(\phi) \subseteq \texttt{inp}(X) \xrightarrow{\phi^{in}} \texttt{int}(\phi) + \texttt{inp}(X)$$

c. Composition formula

4. Tensor product formula

III. The algebra of propagators

A.

IV. Applications of this kind of setup

V. Relationship to string diagrams and traced monoidal categories