Categorical databases

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Purpose of the talk

- We live in a world of interacting entities.
  - Humans interact with machines.
  - Companies interact with companies.
  - Databases interact with applications.
  - And so on, ad infinitum.
- Each entity has its own internal language optimized for operating in some context.
- Interaction between entities is mediated by an interaction between their languages.
- Communication is the successful transfer of information through interaction.
- We need to find a formal underpinning for meaningful communication.

Databases as languages

- A database schema serves as a language, optimized for the operation of some entity in some context.
- But today, entities and contexts change more and more frequently.
- We need to transfer information not only to others, but to our later, different selves.
- We should think of data management as one simple idea: a translation of information from one form to another.
  - An ETL process translates from normalized form to warehouse form.
  - A data migration between databases translates from one schematic form to another.
  - A query against a database translates from all-purpose form to specific-purpose form.
  - An update does not change the schematic form, but still procedurally translates information.

Category theory for managing change of form

There is an fundamental connection between databases and categories.

I propose that:

- Category theory can simplify how we think about and use databases.
- We can clearly see all the working parts and how they fit together.
- Powerful theorems can be brought to bear on classical DB problems.

The pros and cons of relational databases

- Relational databases are reliable, scalable, and popular.
- They are provably reliable to the extent that they strictly adhere to the underlying mathematics.
- Make a distinction between
  - the system you know and love, *vs.*
  - the relational model, as a mathematical foundation for this system.

You're not really using the relational model.

- Current implementations have departed from the strict relational formalism:
  - Tables may not be relational (duplicates, e.g from a query).
  - Nulls (and labeled nulls) are commonly used.
  - Even updates are outside the model.
- The theory of relations (150 years old) is not adequate to mathematically describe modern DBMS.
- The relational model is either clumsy or completely absent in describing:
  - Foreign keys,
  - Updates,
  - Schema mappings and data migration,
  - Distributed databases, etc.
- Databases have been intuitively moving toward what's best described with a more modern mathematical foundation.

Category theory gives better description

- Category theory (CT) does a better job of describing what's already being done in DBMS.
  - Puts functional dependencies and foreign keys front and center.
  - Allows non-relational tables (e.g. duplicates in a query).
  - Labeled nulls and semi-structured data fit in neatly.
- CT offers guidance for all sorts of information hand-off:
  - Foreign key as a handoff from one table to another.
  - Query as a handoff from general schema to specific schema.
  - ETL as a handoff from normalized schema to unnormalized schema.
  - Data migration as information handoff from one schema to another.
  - Update as information handoff from a schema to itself.
  - Applications as information handoff between programming language and database.

What is category theory?

- Category theory is the mathematics of information handoff.
  - That is, CT is about integrity of relationship under change of form.
- Since its invention in the early 1940s, category theory has revolutionized math.
- It's like set theory and logic, except less floppy, more principles-based.
- Category theory has been proposed as a new foundation for mathematics (to replace set theory).
- It was invented to build bridges between disparate branches of math by distilling the essence of mathematical structure.

Branching out

- Category theory naturally fosters connections between disparate fields.
- It has branched out of math and into physics, linguistics, and materials science.
- It has had much success in the theory of programming languages.
- The pure category-theoretic concept of *monads* has vastly extended the reach of functional programming.
- Can category theory improve how we think about databases?

Schemas are categories, categories are schemas

- The connection between databases and categories is simple and strong.
- Reason: categories and database schemas do the same thing.
  - A schema gives a framework for modeling a situation;
    - Tables
    - Attributes
  - This is precisely what a category does.
    - Objects
    - Arrows.
  - They both model how entities within a given context interact.
- The functorial data model is what you get when you demand:

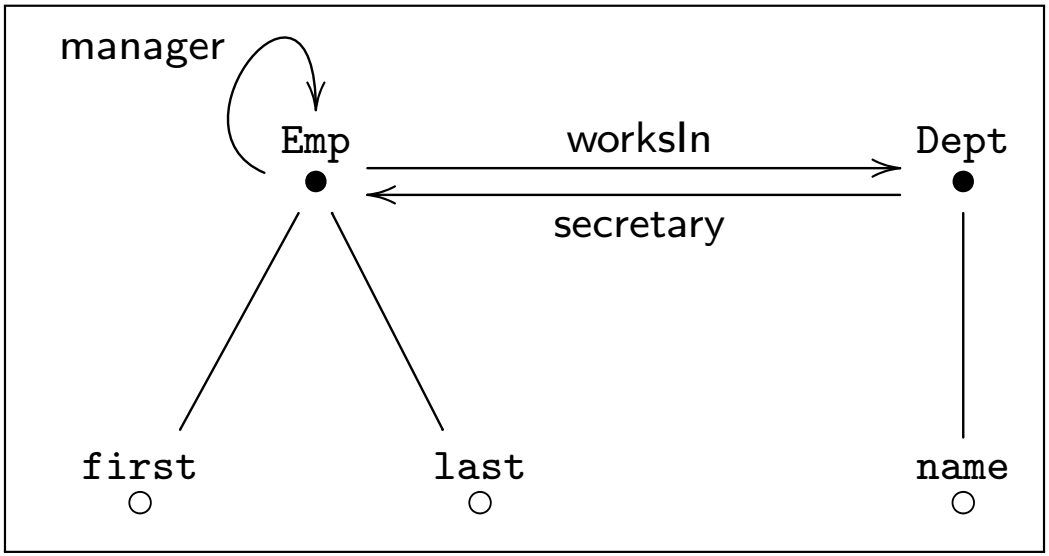
$$\text{Schema} = \text{Category}.$$

The category of categories

- A database schema can be modeled as a finite category.
  - Both model the interactions between tables (via columns).
  - The semantics of this category is captured by instances.
  - Each instance describes the relationship between rows in different tables.
- But there is also a *category of categories*.
  - This models the interactions between database schemas.
  - The semantics of this category is understood as data migration, querying etc.
  - Data migration describes the relationship between instances on different schemas.
- Instead of describing all this today, let's just work on the basic idea.
  - My colleague believed it would be better to demo the tool than get into the math.
  - I'll be available for questions later, and there are papers online.

The basic idea

- In the **functorial data model**, a database schema is a special kind of entity-relationship (ER) diagram.

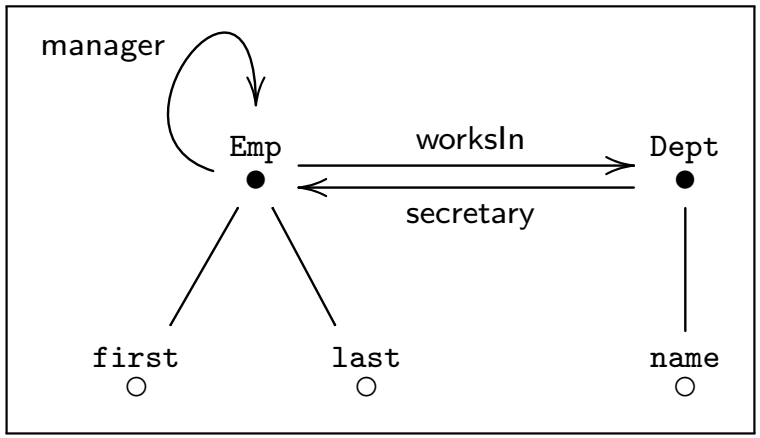


$\texttt{Emp.manager.worksIn} = \texttt{Emp.worksIn}$ $\qquad$ $\texttt{Dept.secretary.worksIn} = \texttt{Dept}$

Emp				
Emp	**mgr**	**works**	**first**	**last**
101	103	q10	Al	Akin
102	102	x02	Bob	Bo
103	103	q10	Carl	Cork

Dept		
Dept	**sec**	**name**
q10	102	CS
x02	101	Math

The basic idea, continued



- Each node represents an entity set.
- Each entity is identified by a globally unique ID and has some attributes (strings, integers, etc).
- Each directed edge represents a foreign key.
- Data integrity constraints are path equalities.
- Instances are **always and only** considered up to isomorphism of IDs and equality of attributes.
- No nulls.
- We usually think of each attribute and edge as a binary table.

Outline

- The functorial data model can be studied with the regular tools of relational database theory.
- The functorial data model can also be studied with **category theory**.
  - Category theory reveals many additional useful properties of functorial schemas and instances that are invisible to traditional database theory.
- The purpose of this talk is to demonstrate these properties without getting into the mathematics of category theory.
- To do this, we will describe FQL, a functorial query language, and the relationship between FQL and SQL.
- Everything in this talk has been implemented by Ryan Wisnesky.
  - Download the FQL IDE from: `http://categoricaldata.net/fql.html`

Schema mappings and associated operations

- A **schema mapping** $F : S \to T$ is a constraint-respecting mapping:

$$nodes(S) \to nodes(T) \qquad edges(S) \to paths(T)$$

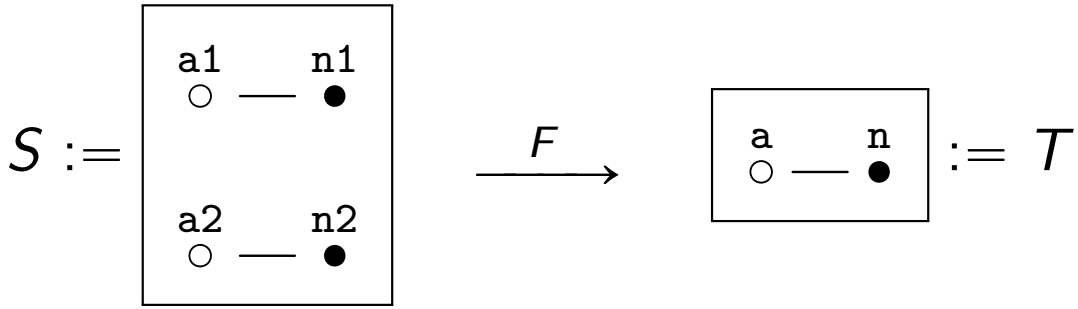
and it induces three **data migration** operations:

- $\Delta_F : T - inst \to S - inst$ (like projection)
- $\Sigma_F : S - inst \to T - inst$ (like union)
- $\Pi_F : S - inst \to T - inst$ (like join)

Demos

- I'll give some examples and then demo them in the FQL IDE.
- In each case, I'll show a couple FQL schemas $\mathbf{S}, \mathbf{T}$ and an FQL mapping $F : \mathbf{S} \to \mathbf{T}$ between them.
  - I'll talk about what the three data migration functors, $\Delta_F, \Sigma_F, \Pi_F$ do in each case.
  - I'll show some EDs (embedded dependencies) that would have the same result under the chase.

Example 1



- Δ_F : T–**Inst** $\to$ S–**Inst** copies n into n1 and n2, and a into a1 and a2.

$$\mathtt{a}(x,y) \to \mathtt{a1}(x,y) \wedge \mathtt{a2}(x,y)$$

- Π_F : S–**Inst** $\to$ T–**Inst** joins a1 and a2 into a, creating a fresh ID for each tuple.

$$\mathtt{a1}(x,y) \wedge \mathtt{a2}(x',y) \to \exists z, \mathtt{a}(z,y)$$

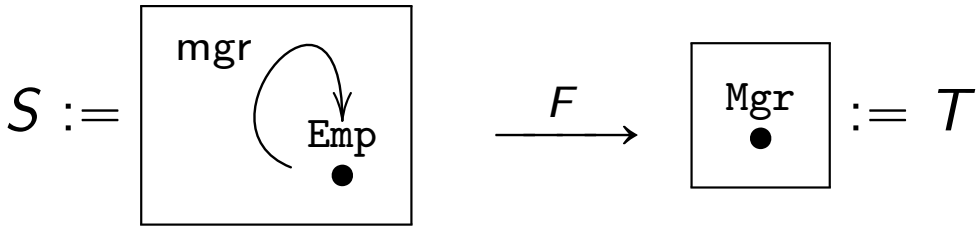
- Σ_F : S–**Inst** $\to$ T–**Inst** unions a1 and a2 into a.

$$\mathtt{a1}(x,y) \to \mathtt{a}(x,y) \qquad \mathtt{a2}(x,y) \to \mathtt{a}(x,y)$$

Demo Example 1 in FQL IDE

```
schema S = {nodes n1, n2; attributes a1 : n1 -> string, a2 : n2 -> string;
  arrows; equations; }

schema T = {nodes n; attributes a : n -> string; arrows; equations;}

mapping F = {
 nodes n1 -> n, n2 -> n;
 attributes a1 -> a, a2 -> a;
 arrows; } : S -> T

instance I = {
 nodes n1 -> {0,1,2}, n2 -> {3,4};
 attributes a1 -> {(0,alpha),(1,beta),(2,gamma)},
   a2 -> {(3,alpha),(4,upsilon)};
 arrows; } : S

instance pi_F_I = pi F I
instance sigma_F_I = sigma F I
```

Example 2



- Π_F will migrate into `Mgr` only those `Emps` that are their own `mgr`.
- Σ_F will migrate into `Mgr` the "management groups" of `Emp`, i.e. equivalence classes of `Emps` modulo the equivalence relation generated by `mgr`.
- Key point of the examples: Functorial data migration operators are very expressive.
  - Note that none of these examples used path equality constraints.
  - We can be even more expressive if we employ them.

FQL - A Functorial Query Language

- Functorial data migrations have a useful normal form:

$$\Sigma_F \circ \Pi_{F'} \circ \Delta_{F''}$$

- Caveat: F must obey a restriction that (roughly) it only takes unions of tables that are "union compatible."
- We call data migrations above the above form **FQL queries**.
- Analogously, unions of conjunctive queries are a useful normal form for relational algebra.

Key results:

FQL queries can all be written in the following form:

$$\Sigma_F \circ \Pi_{F'} \circ \Delta_{F''}$$

- FQL queries are closed under composition.
  - Meaning we can implement compositions without materializing intermediate results.
- Unions of conjunctive queries can be implemented in FQL.
  - Natively it has bag semantics, which can be useful.
  - One can also obtain set semantics, after some simple post-processing.
- Every FQL query can be implemented as a union of conjunctive queries under set semantics
  - Note that we need an operation for creating globally unique IDs (e.g., using SQL auto-generated row IDs).

Demo - People

- As you can see, the FQL IDE generates SQL, displayed at the bottom.
  - Note that the FQL IDE is executing that SQL via JDBC on a 3rd-party SQL engine.
- We can also see an operation category theory produces for free, the category of elements.
  - This operation is interesting because it converts any instance to a schema.
  - You can also see it as producing an RDF triple store.
  - Time permitting, I'll discuss this at the end of the talk.

Demo - RA to FQL

- Let's look at how to translate unions of conjunctive queries (SPCU) to FQL.
  - The SELECT and FROM clauses are Δ, which gathers the required tables.
  - the WHERE clause is Π, which joins.
  - the UNION clause (bag semantics) is Σ, which takes unions.
  - (For set semantics, we post-process with something called RELATIONALIZE.)
- Show the active domain, and remark that it is expensive to compute.
- We can also translate SQL schemas to FQL
  - Each table must have a single primary key column.
  - Any number of foreign key constraints.

FQL vs SQL

Since FQL compiles to SQL, why not just write in SQL?

- FQL query results are entire databases, complete with foreign keys.
  - SQL only computes one table at a time.
- With FQL, output databases are guaranteed to obey their path-equality constraints.
  - One can write SQL queries that do not satisfy the necessary constraints.
  - FQL uses the foreign-key architecture of the target schema as part of its query semantics.

Since SQL compiles to FQL, why not just write in FQL?

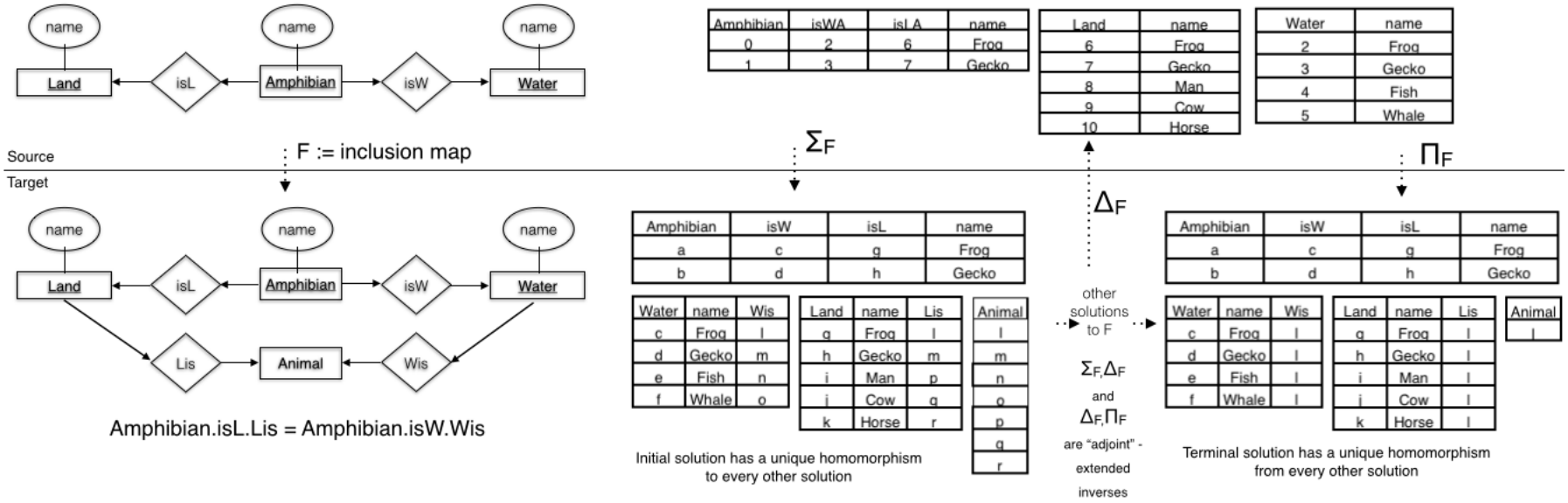
- Currently, schemas and instances imported from SQL are not appropriately native to FQL.
  - We saw that we can compute unions of conjunctive queries, so it's not bad.
  - But to get the correct behavior, they are encoded using an expensive "active domain" construction that is not feasible in practice.

FQL and Data Exchange

- When the F in Σ_F is not a "discrete op-fibration", Σ_F cannot be computed by SQL.
- Key result: it can be computed by chasing a set of embedded dependencies.
- The semantics of such Σ_F is similar to that of Clio or other data exchange systems, but
  - Σ_F has better properties (e.g., closure under composition)
  - Σ_F is more powerful (e.g., can compute connected components of a graph)

Demo: Data Exchange

- I will demo an example in which we union along foreign keys.
  - Amphibians has a foreign key to land animals and to water animals.
  - We add a new table (animals) and a new path equation.
  - FQL generates some EDs.
  - We get the right number of animals (7).
  - Clio computes 9 animals: it ignores the path equality constraint, because it only handles TGDs.



How's the time?

Shall we skip to the summary now, or keep going with RDF?

FQL to RDF

- Category theory has an operation for converting any instance into a schema.
  - Since all schemas are graphs, what graph do you get?
  - Answer: the RDF graph.
- We have FQL emitting OWL and RDF.
  - Functorial schemas (without path equations) can be output as OWL.
  - Every functorial instance is naturally encoded as RDF.
  - And of course, the RDF is verified against its OWL schema.
- Demo: Employees example
  - Note the RDF and OWL output.
  - Note the category of elements.

Summary

- By restricting to functorial schemas and instances, we gain many useful properties:
  - Schemas
    - are ER diagrams
    - have data integrity constraints built-in
  - Data migrations
    - are weak inverses to each other
    - operate on entire databases
    - preserve constraints
    - can implement unions of conjunctive queries
    - are closed under composition (exception: "special Σ")
    - are implementable in SQL (exception: "special Σ")
  - All these properties were discovered through category theory.

Status

- Current work:
  - The view-update problem for FQL.
  - Additional programming constructs for FQL (e.g., exponentials).
- Future work:
  - Integrating FQL with a general-purpose programming language.
  - A "native", non-SQL implementation of FQL.
  - Grouping, aggregation, nesting, difference, nulls.
  - Optimization.

Thanks

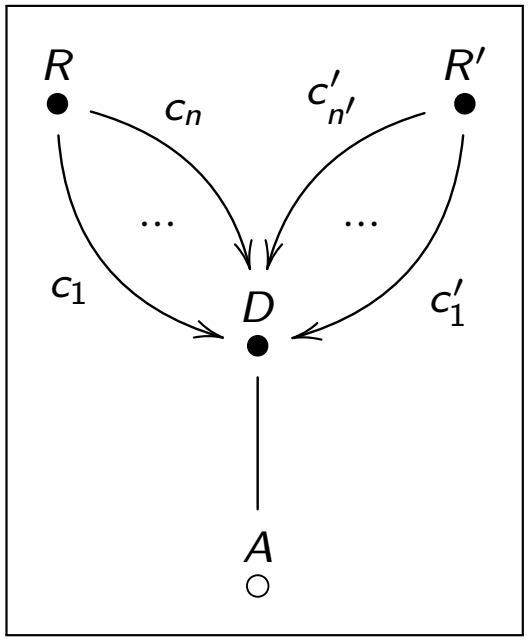
Thanks for inviting me to speak!

Programming in FQL

- The functorial data model admits many useful operations on schemas and mappings: products, co-products, etc.
- In fact, there is enough structure to interpret the simply-typed λ-calculus (STLC).
  - Key result: every type in the STLC denotes a schema, and every (open) term denotes a mapping.
- For each schema S, the functorial data model admits many useful operations on S-instances and S-homomorphisms: products, co-products, etc.
- In fact, there is enough structure to interpret higher-order logic (HOL).
  - Key result: every type in HOL denotes an S-instance, and every (open) term denotes an S-homomorphism.
- Show FQL products and co-products example.

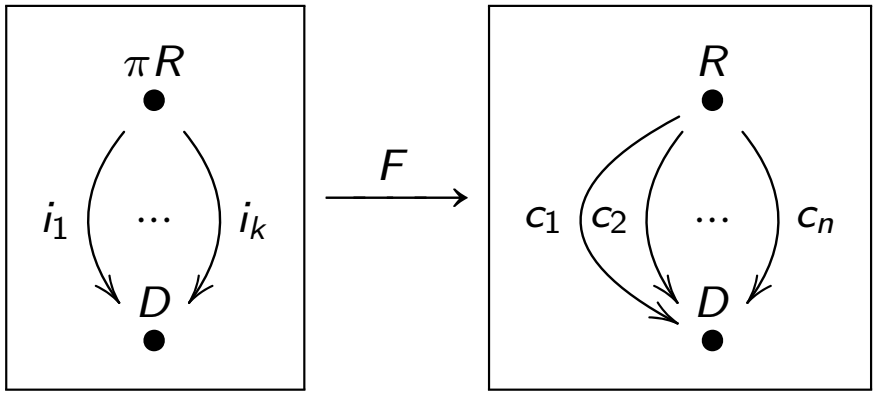
SQL as FQL – the encoding

- We can always encode arbitrary relational databases as functorial instances using an explicit active domain construction. Consider a relational schema with two relations $R(c_1, \ldots, c_n)$ and $R'(c'_1, \ldots, c'_{n'})$



SQL as FQL – Projection

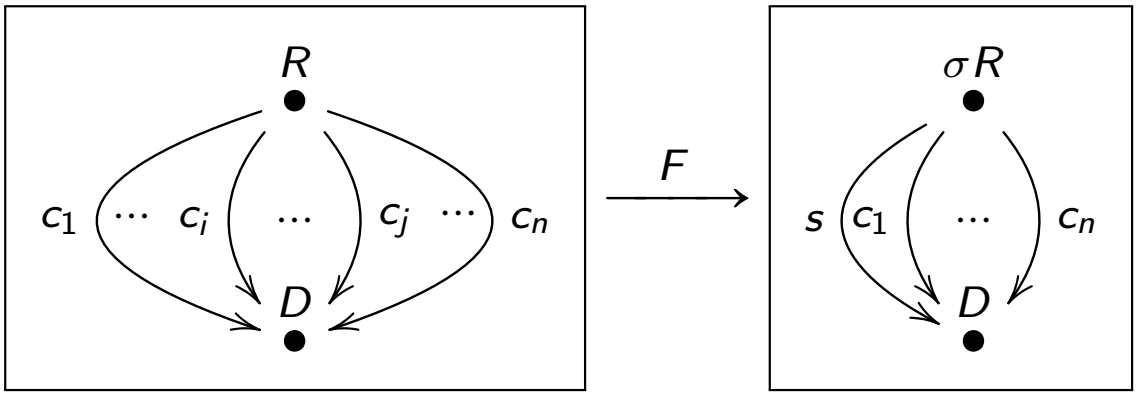
- Let R be a table. We can express $\pi_{i_1,\ldots,i_k} R$ using Δ_F



This construction is only appropriate for bag semantics because πR will will have the same number of rows as R.

SQL as FQL – Selection

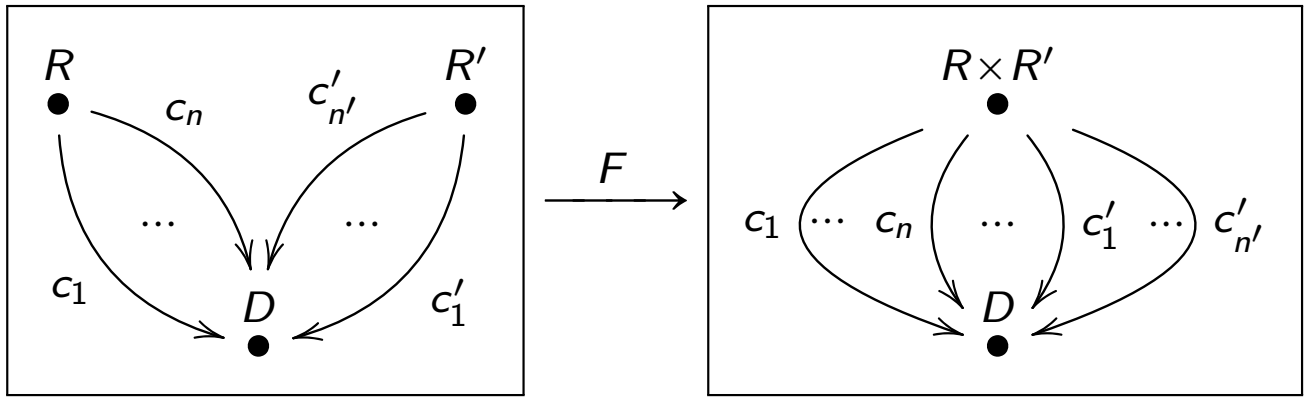
- Let R be a table. We can express $\sigma_{i=j}R$ using Π_F:



Here $F(c_i) = F(c_j) = s$.

SQL as FQL – Product

- Let R and R' be tables. We can express $R \times R'$ as Π_F



FQL as SQL – implementing the data migration functors

- Δ can be implemented with conjunctive queries and ID-generation.
  - Target node tables are copied from the source. Target edge/attribute tables are populated by compositions of source edge/attribute tables. ID-generation only used to restore globally unique ID requirement.
- Σ can be implemented with unions of conjunctive queries and ID-generation.
  - Algorithm is similar to a "union of Δs".
- Π can be implemented with conjunctive queries and ID-generation.
  - The most difficult to implement. Requires computing large "limit" tables that are similar to "join all'. ID-generation used to create IDs for the rows in the limit tables.