CATEGORY THEORY – A FORMAL AND FLEXIBLE INTERDISCIPLINARY LANGUAGE

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1. Talk

1.1. **Summary.**

(1) Thanks
(2) CT–what is it, history.
(3) Disciplines, separation, connection.
(4) Interdisciplinary communication.
(5) Categories and functors.
(6) CT in math today. Can it function similarly in science?

1.2. **Details.**

(1) Thanks to Piet. Great to be at IAS!!
(2) CT–what is it, history.
  (a) CT is a field of math like math is a field of science — queen and servant.
  (b) History—algebraic topology.
(3) Disciplines, separation, connection.
  (a) Discipline: perspective; notion of correctness; forms, legal moves, and sameness of outcome.
  (b) Separation: buffer allows a consistent practice by which methods are optimized.
  (c) Connection: if information can be transferred, problems can be solved by many disciplines running in parallel or series, rather than only by one discipline.
(4) Interdisciplinary communication. The ideal language should be agnostic about the meaning, caring instead about faithful preservation of forms and legal manipulations.
(5) Categories and functors.
  (a) A category is the mathematical formulation of a discipline: forms (objects), legal moves (morphisms), sameness of outcome (commutative diagrams).
  (b) Each category is separated as its own thing. This way it can be studied in its own right.
  (c) But categories are connected by functors, so that one can reason in multiple domains and preserve truth as information is passed between them.
(6) CT in math today.
  (a) CT is dominating math today. We're on the steep part (or beyond) on the logistic adoption curve.
  (b) It is innervating math.
  (c) I'm betting it can function similarly in science.

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2. Preliminary ideas

- Q: What makes a language good for interdisciplinary communication?
    - Q: What is the purpose or value of interdisciplinary communication?
        * Q: What is a discipline?
        * T: Some examples of disciplines: math, CS, bio. Some qualities of disciplines:
            · a certain basic canon of knowledge;
            · a way to answer questions independently from others and come up with the same answers;
            · a recognition of correctness;
            · a certain mode of thinking, a perspective, a style of looking, a style of interpretation;
            · a way of moving from problem to solution;
            · techniques for investigating and for breaking off pieces of a problem.
        * Q: Why are disciplines separated?
        * T: Disciplines may be separated to allow optimization of language and technique. Practice with manipulating the forms requires repetition, which in turn requires a relatively stationary and committed perspective.
        * Q: Why should disciplines be connected?
        * T: A discipline survives because it is perceived to provide something, to solve a kind of problem. But the problem doesn't exist in a vacuum. Disciplines have interfaces through which problems are input and solutions are output. But these solutions may then be submitted as problems to other disciplines. Disciplines are connected for the success of the larger structure.
        * Summary: A discipline is a set of recognizable forms and a set of agreed-upon steps, legal moves, that allow you to proceed from one form to another. A discipline is kept distinct so that a practice can develop to repeatedly manipulate forms without continually changing perspective. In this way, maneuvering within it can be optimized. Disciplines are then connected to solve larger problems by handing off a solution from one perspective as a problem submitted to another perspective.
    - A: From the perspective of the larger whole, disciplines should be connected, meaning their terminology should be translated in ways that facilitate transmission of problems and solutions, because in this way we have the possibility of applying any number of disciplines to work on the same problem, rather than being confined to just one.
- T: A good interdisciplinary language should be agnostic when it comes to interpretation of forms. It should encode the notion of form and the notion of "legal move" and of coincidence of outcome. A translation from one discipline to another involves a relationship between their respective forms and between their respective legal moves, such that coincidence of outcome is preserved.