# Category theory, the theory of mathematical structures: a whirlwind tour

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**Abstract**

Category theory has become a fundamental aspect of modern research in pure mathematics, and it is branching out to other disciplines as well, including physics and computer science. In this talk I will present category theory as the theory of mathematical structures. I will discuss a range of examples, showing how category theory highlights the tightly-interwoven structure of mathematics.

**I. Introduction**

- **A. Category theory's current place in mathematics and beyond**
- **B. What it's about—classifying types of structures**
- **C. Like all math, it's partly a language: one you can trust.**
- **D. It's very abstract, allowing efficient communication of big thoughts.**

**II. Wading in with matrices**

- **A. Invertible matrices – group**
- **B.** $n \times n$**-matrices – monoid**
- **C.** $n \times p$**-matrices – category**
- **D. The category** $\mathsf{Vect}$

**III. Let's do it again with sets**

- **A. Sets and functions**
- **B. Automorphisms and endomorphisms**
- **C. The category** $\mathsf{Set}$

D. Commutative diagrams in Set

E. Products and coproducts in Set

F. Vector spaces reprise:

1. Commutative diagrams in Vect
2. Products and coproducts in Vect

IV. Categories and functors

A. Definition of category

B. Definition of functor

C. Examples of categories and functors

1. sets, vector spaces (functors between them)
2. preorders and sets (functors between them)
3. monoids, groups, rings (functors between them)
4. most important thing I'm not telling you: adjunctions

D. Cat as category

E. Some special categories and functors between them

1. preorders as special categories
2. monoids and groups as special categories
3. discrete dynamical systems
4. group representations as functors