

Notes for LambdaConf

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Abstract

In the first two-hour block, we will discuss categories, focusing on the category of sets and functions, which is a reasonably good model for thinking about datatypes and programs. We'll discuss products, coproducts, exponential objects, and more. Then we'll define abstract categories and discuss examples such as posets, monoids, and finitely presented categories.

In the second two-hour block, we'll discuss functors, natural transformations, and adjunctions, as well as their relationship to the examples given so far, and with a view toward some monads that occur in functional programming.

In the third two-hour block, we'll discuss monads, their Kleisli categories, and briefly touch on initial algebras for endofunctors. We'll also discuss monoidal categories and their string diagrams, including traced monoidal categories (which I hear has some relation with functional reactive programming). We will also briefly discuss operads and multisorted equational theories, as well as how to view string diagrams in this context.

Consider pairing academics with industry people.
Set a timer for each section.

Chapter 1

Introducing categories

1.1 Sets and functions

- 05 mins. (Interspersed questions throughout.)
 - 05 mins. Sets and functions (plus identity and composition)
 - 05 mins. Isomorphisms
 - 05 mins. Terminal and initial sets, $\text{Hom}(*, -)$.
 - 10 mins. Products and their universal property
 - 05 mins. (Participants talk to their neighbor.)
 - 05 mins. Coproducts and their universal property
 - 05 mins. Exponential objects; currying
 - 05 mins. Arithmetic
 - 05 mins. Arithmetic shows the power of universal definitions.
 - 05 mins. (Participants talk to their neighbor.)
 - 05 mins. The limit of any finite diagram in **Set**
 - 05 mins. Break
- ~ 70 minutes.

1.2 Categories

- 05 mins. (Interspersed questions throughout)
 - 05 mins. Definition of category
 - 05 mins. The category **Set** vs. discrete categories
 - 05 mins. Posets
 - 05 mins. (Participants talk to their neighbor.)
 - 05 mins. Datatypes and programs
 - 05 mins. Monoids
 - 10 mins. Finitely-presented categories, briefly discuss FQL.
- ~ 45 minutes.

Chapter 2

Functors, natural transformations, and adjunctions

2.1 Functors

05 mins. (Interspersed questions throughout)

05 mins. Definition of functor F (plus id_C and $F; G$).

02 mins. Terminal and initial category, functors $* \rightarrow C$.

03 mins. Full, faithful functors. (Lots of faithfuls; "inhabited" $\text{Set} \rightarrow \{\emptyset, \{*\}\} \rightarrow \text{Set}$ is full)

05 mins. **Cat** and full subcats **Set**, **Poset**, **Monoid**.

05 mins. (Participants talk to their neighbor.)

10 mins. Directed graphs, $\text{Fun}(\boxed{\bullet \rightrightarrows \bullet}, \text{Set})$

05 mins. (Participants work out monoid actions.)

10 mins. Categorical databases

05 mins. Break.

~ 55 minutes

2.2 Natural transformations

05 mins. (Interspersed questions throughout)

05 mins. Map of directed graphs.

05 mins. Definition of natural transformation.

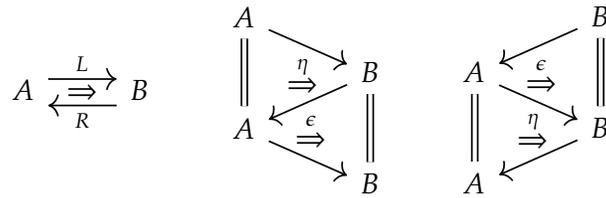
05 mins. The category of functors $\text{Fun}(C, D)$

05 mins. (Participants work out case for $\text{Fun}(\boxed{\bullet \rightrightarrows \bullet}, \text{Set})$ and $\text{Fun}(*, C)$.)

05 mins. C-shaped diagrams in Set .

~ 30 mins.

2.3 Adjunctions



05 mins. (Interspersed questions throughout)

02 mins. Hom-set definition of adjunction

03 mins. Unit and counit

05 mins. Alternate definition from unit/counit

05 mins. Galois connections. E.g. for any $0 < x$ and $a, b \in \mathbb{N}$, $\lceil a * x \rceil \leq b \Leftrightarrow a \leq \lfloor b/x \rfloor$

10 mins. Examples: 1. **Set** \Leftrightarrow **Monoid** 2. Currying adjunction, 3. **Set** $\xrightarrow{+1} \mathbf{Set}_{\text{par}}$

05 mins. Left adjoints preserve colimits, right adjoints preserve limits.

~ 35 mins.

Chapter 3

Monads, algebras, and monoidal categories

3.1 Monads and algebras

05 mins. (Interspersed questions throughout)

05 mins. Monads on C .

05 mins. Every adjunction has a monad.

05 mins. Kleisli category of a monad.

10 mins. The three examples above: List, State, Maybe.

05 mins. (Participants talk to their neighbor.)

05 mins. The Eilenberg-Moore category of a monad

05 mins. Algebras for arbitrary endofunctors; initial algebras.

05 mins. Example: Lists on A .

05 mins. (Participants find the endofunctor for binary trees.)

05 mins. Break.

~ 60 mins

3.2 Monoidal categories

05 mins. (Interspersed questions throughout)

05 mins. Definition of strict monoidal structure

05 mins. How usual definition differs (bookkeeping).

05 mins. Example: cartesian monoidal structure.

05 mins. (Participants discuss cocartesian monoidal structure.)

10 mins. String diagrams

05 mins. Traced monoidal categories

05 mins. Cartesian operads

05 mins. Multisorted equational theories

05 mins. Operads of string diagrams

05 mins. Examples: categories, monoidal categories, traced categories

~ 55 mins