Metric realization of fuzzy simplicial sets

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Abstract

Abstract: We will discuss how the usual story of simplicial sets and their geometric realization as topological spaces can be repeated for fuzzy simplicial sets. In particular, we will construct a metric realization functor with a right adjoint sending every metric space to a fuzzy simplicial set.

Metric realization of fuzzy simplicial sets

I. Introduction

A. Goal: prepare for talk on dimension reduction application

B. Simplicial sets (combinatorial) and topological spaces (spatial)

C. Fuzzy simplicial sets and metric spaces

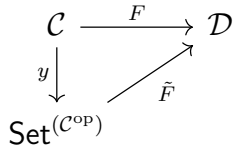
II. Simplicial sets

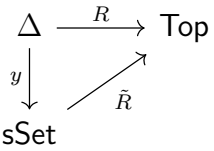
A. Intro

1. Graphs and reflexive graphs
2. Higher simplices
3. Functor $R\colon \Delta \to \mathsf{Top}$

B. Simplicial vs. symmetric simplicial

C. Geometric realization (Yoneda extension) and singular functor





III. Fuzzy sets and simplicial sets

A. Classical fuzzy sets and morphisms (Zadeh)

B. As sheaves on $[0, 1]$. (Barr) (Has powersets; classical fuzzy sets don't)

C. Fuzzy simplicial sets: sheaves on $\Delta \times [0, 1]$

IV. Metric realization

A. Metric spaces

1. Various flavors (dagger, skeletal today)
2. Morphisms: non-expansive maps
3. Need colimits.

B. Functor $\Delta_F \to \mathsf{Met}$

1. $-\log\colon [0, 1] \cong [0, \infty]$
2. For $d \in [0, \infty]$, put $\Delta^n_d \mapsto \{(x_0, \ldots, x_n) \in \mathbb{R}^{n+1}_{\geq 0} \mid x_0 + \cdots + x_n = d\}$

C. The geometric realization functor

$$\mathsf{sSet}_{\mathsf{Fuz}} \leftrightarrows \mathsf{Met}$$

1. Note: colimits are much easier to compute in $\mathsf{sSet}_{\mathsf{Fuz}}$