

Graphical presentations of abelian categories

David

August 27, 2019

Graphical presentations of abelian categories

time check: 2:30

I. Introduction

- A. Show Kernel and cokernel,
- B. Show breaking, merging, abelian logic

II. Supply

- A. Intro: common theme 1 min
- B. Definitions 9 mins
 - 1. Supply
 - 2. Homomorphisms
 - 3. Po-supply
 - 4. Lax homomorphisms
 - 5. Homomorphic supply
- C. Examples 4 mins
 - 1. Categories with products: homomorphic supply of comonoids ($\text{FinSet}^{\text{op}}$)
 - 2. Self-dual compact closed categories: supply of unoriented Cob
 - 3. Hypergraph categories: supply of Cospan
- D. Theory 3 mins
 - 1. \mathbb{P} supplies \mathbb{P}
 - 2. Change of supply
 - 3. Induced map $\bigsqcup_{c \in \text{Ob}(\mathcal{C})} \mathbb{P} \rightarrow \mathcal{C}$

time check: 2:47

III. Relational po-categories

- A. Regular categories 5 mins
 - 1. Extremal epis
 - 2. Image factorization
 - 3. Definition
 - 4. Assumptions:
 - a. Have cartesian monoidal structure $\times, 1$ (Makkai anamonical cats)

b. Well-powered.

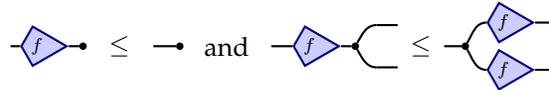
B. Relational po-categories

1. Definition

5 mins

a. Monoidal po-category

b. Supplies $\text{Cospan}^{\text{co}}$ and every map is lax comonoid hom.

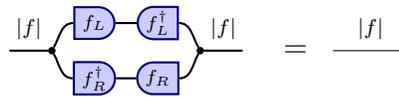


c. Tabulations: every map $f: r \rightarrow s$ can be written as $f_R \circ f_L$ with

(1) $f_R: r \rightarrow |f|$ right adjoint

(2) $f_L: |f| \rightarrow s$ left adjoint

(3) $\delta_{|f|} \circ (f_R^\dagger \otimes f_L) \circ (f_R \otimes f_L^\dagger) \otimes \mu_{|f|} = \text{id}_{|f|}$,



2. 1-morphisms: strong monoidal functors.

1 min

3. 2-morphisms: left adjoints in the functor po-category $[\mathbb{R}, \mathbb{R}']$

2 mins

C. Equivalence

1. Relations $\text{Rel}: \mathcal{RgCat} \rightarrow \mathcal{RlCat}$

2 mins

2. Left adjoints $\text{LAdj}: \mathcal{RlCat} \rightarrow \mathcal{RgCat}$

3 mins

a. Draw pullbacks

b. Epi-mono factorization of $f: r \rightarrow s$

(1) tabulate $\eta_r \circ f$ as $1 \xleftarrow{!} \text{im}(f) \xrightarrow{L} s$

(2) Then $f \circ L^\dagger$ is epi and L is mono.

3. 2-equivalence

1 min

4. Descends to 1-equivalence

1 min

a. For any \mathcal{R} have io ff functor $\mathcal{R} \rightarrow \text{LAdj} \circ \text{Rel}(\mathcal{R})$

b. For any \mathbb{R} have io ff functor $\mathbb{R} \rightarrow \text{Rel} \circ \text{LAdj}(\mathbb{R})$.

time check: 3:07

IV. Regular and abelian calculi

A. Goal: separate syntax from semantics

1. Regular logic

- a. Multiple variables
 - b. $\exists, \wedge, \text{true}, =$.
 - c. \vdash relationship
2. Abelian logic: add \vee, \perp .

Think about
this