Categorical Databases

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Outline

An invitation to engage with us, and solve real-world problems.

A road to true interdisciplinarity

- Scientific disciplines are conceptual analogies of the world.
  - Science: a schematic, conceptual account of phenomena.
  - Engineering is using these accounts to channel world events.
  - But how do different disciplines and accounts cohere?
  - To solve big problems, we need to connect different approaches.

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- We need a shared fabric, a substrate for interdiscipinarity.
  - Interdisciplinarity consists of effective analogy-making.
  - To go further, we need to formalize the analogies themselves.
- Better yet: we need a conceptual stem-cell.
  - Something that can differentiate into huge variety of forms.
  - Find the analogies between forms as aspects within the stem cell.

Category theory as conceptual stem-cell

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Category theory (CT) can differentiate into many forms:

- All forms of pure math... (we'll briefly discuss this)
- Databases and knowledge representation (categories and functors)
- Functional programming languages (cartesian closed categories)
- Universal algebra (finite-product categories)
- Dynamical systems and fractals (operad-algebras, co-algebras)
- Shannon Entropy (operad of simplices)
- Partially-ordered sets and metric spaces (enriched categories)
- Higher order logic (toposes = categories of sheaves)
- Measurements of diversity in populations (magnitude of categories)
- Collaborative design (enriched categories and profunctors)
- Petri nets and chemical reaction networks (monoidal categories)
- Quantum processes and NLP (compact closed categories)

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  - Conclude that math/CT explains everything and hence nothing?

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  - CT—like math—explains, models, formalizes many many things.
  - Conclude that math/CT explains everything and hence nothing?
- Stem cells don't do work until they differentiate.
  - "Adult-level" work requires differentiation and optimization.
  - But the unified origins lead to impressive interoperability.

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You could also say: CT is mathematics, self-aware.

- Designed to transport theorems from one area of math to another.
  - From topology (shapes) to algebra (equations).
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    - It's become a gateway to pure mathematics.
- And it's branched out from math in a big way.
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If you care about information hygiene, CT needs to be on your radar.

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The category-theoretic stem cell is about compositional design patterns.

Let's focus on one: Data frameworks and data transformations.

- The problem: multiple models of similar information
- What is "model-space"?
- Category theory offers a mathematical notion of model-space.
- The kinematics of data: how it moves and rests.

Plan of the talk

- The problem: pervasive and insidious.
- The math: Category theory describes model space.
- The tool: Open-source implementation and commercialization.

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The Copernican revolution continues

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- But there isn't just one best coordinate system.
- Each coordinate system is a perspective, a basis for calculation.

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- Linear Algebra studies coordinate systems *and transformations*.
- But people still search for the "best" information model.
  - E.g. OMOP in EMRs
  - BFO, CIDOC, SUMO, etc., etc. in upper ontologies
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Multiplicity of perspectives is not going away. Let's learn to integrate.

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  - Finding common structures.
- Information kinematics:
  - Information rests in databases.
  - Information moves by data transformations.
  - Let's dig in.

Information kinematics

Information rests primarily in databases.

- Domain knowledge informs the structure of the database.
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- Different situations require different data structures.
- Most common transformation: *querying*:
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  - The result of a query is a schema with only one table.
- Other transformations: ETL, schema evolution, warehousing

Think vector spaces and linear transformations.

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- Each table represents some sort of *entity*:
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  - Its columns represents aspects of that entity.
- Example: name and owner are aspects of a house-cat.
  - The house-cat is an entity.
  - The house-cat table has a name column.
  - The house-cat table has an owner column.
  - A house-cat owner is a person, an entity of type person.

House-cat	Name	Owner
C101	Prince Charming	P52
C241	Patches	P52
C468	Mittens	P81

Person	Name
P17	Alice
P52	Bob
P81	Carl

The house-cat schema

Domain knowledge:

- House-cats have names and owners;
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The database collects worldly examples of this knowledge:

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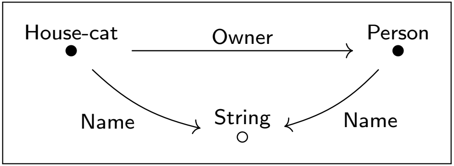
String
Mittens
Patches
⋮

The schema for this knowledge can be drawn as a graph:

$S :=$

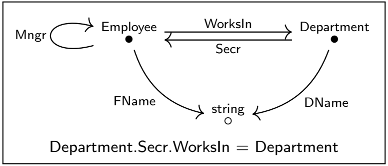


Each column connects its table to another "foreign" table.

A bit more interesting

Let's add loops and integrity constraints:

mySchema :=



Employee	FName	WorksIn	Mngr
1	Alan	101	2
2	Ruth	101	2
3	Kris	102	3

Department	DName	Secr
101	Sales	1
102	IT	3

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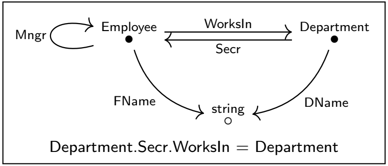

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Stats:

- Three dots, three tables, three ID columns.
- Five arrows, five non-ID columns.

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FOR    e:Employee
WHERE  e.FName = Alice
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- It's a way of transforming data: form A to form B:

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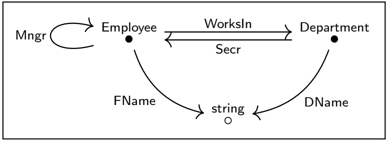
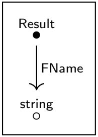


$B :=$



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  - Calculus and diff. eq.? We can hire people to do that.
  - But 40% of IT budgets are spent on information integration.
  - We're constantly breaking and reviving Humpty Dumpty.
  - IT culture has a poor understanding of data transformations.
  - IT culture doesn't even seem to name this problem explicitly.

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$$src, tgt\colon\ \mathrm{Arr}(\mathcal{C}) \to \mathrm{Ob}(\mathcal{C}),$$

   assigning to each arrow its *source* and its *target* object, respectively.
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3. An notion of equivalence for paths, denoted $\simeq$.

Definition of a category II: Rules

These constituents must satisfy the following requirements:

1. If $p \simeq q$ are equivalent paths then the sources agree: $src(p) = src(q)$.
2. If $p \simeq q$ are equivalent paths then the targets agree: $tgt(p) = tgt(q)$.

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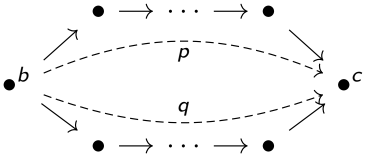

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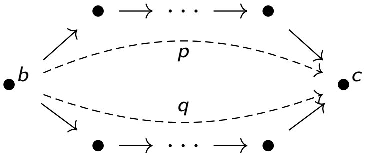
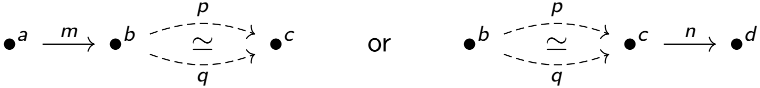


If $p \simeq q$ then for any extensions



We have equivalences: $m \mathbin{⨟} p \simeq m \mathbin{⨟} q$ and $p \mathbin{⨟} n \simeq q \mathbin{⨟} n$.

Categories = database schemas

- Database schemas are categories!
  - The objects of the category $\mathcal{C}$ are tables.
  - The arrows of $\mathcal{C}$ are columns, connecting one table to another.
  - The integrity constraints are path equations.
  - We brush some details under the rug (distinction between $\bullet$ and $\circ$).

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  - We brush some details under the rug.
- But there are also categories that are well-known in math.

Categories from mathematics

Categories are everywhere in mathematics.

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Categories from mathematics

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- The category **Set** of sets:
  - Objects = all sets
  - arrows $S \to T$ = all functions from S to T
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There's also a notion of *mapping* between categories: functors.

Functors: mappings between categories

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- A functor is a graph mapping that respects path equivalence.

- **Definition**: A functor $F\colon \mathcal{C} \to \mathcal{D}$ consists of
  - a function $\text{Ob}(\mathcal{C}) \to \text{Ob}(\mathcal{D})$ and
  - a function $\text{Arr}(\mathcal{C}) \to \mathrm{Path}(\mathcal{D})$,

  such that F
  - respects sources and targets,
  - respects equivalences of paths.

Functors and databases

Recall:

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Recall:

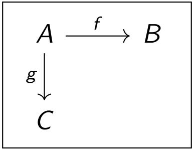
- A category is a directed graph with path equivalences.
- A functor $\mathcal{C} \to \mathcal{D}$ is a mapping that preserves these structures.
- **Set** is a category; recall
  - its objects are all sets
  - its arrows $S \to T$ are functions, and
  - two paths are equivalent if they compose to the same function.

Functors and databases

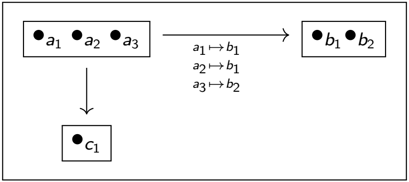
Recall:

- A category is a directed graph with path equivalences.
- A functor $\mathcal{C} \to \mathcal{D}$ is a mapping that preserves these structures.
- **Set** is a category; recall
  - its objects are all sets
  - its arrows $S \to T$ are functions, and
  - two paths are equivalent if they compose to the same function.
- A functor $\mathcal{C} \to$ **Set** fills schema $\mathcal{C}$ with data.
  - Example: Let $\mathcal{C}$ be the category on the left.
  - Then here's an example functor $I : \mathcal{C} \to$ **Set** :

$\mathcal{C} :=$

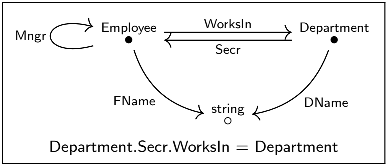


$I :=$



Schema=Category, Instance=Set-valued functor

- Let $\mathcal{C}$ be the following category

$\mathcal{C} :=$ 

- A functor $I \colon \mathcal{C} \to \mathbf{Set}$ consists of
  - A set for each object of $\mathcal{C}$ and
  - a function for each arrow of $\mathcal{C}$, such that
  - the declared equations hold.
- In other words, I fills the schema with compatible data.

Summary of the connection

- The connection between categories and databases is simple.
- A database schema is a custom category.
- Functors $I: \mathcal{C} \to \mathbf{Set}$ are database instances.
- What about functors $F: \mathcal{C} \to \mathcal{D}$ between schemas?

Data transformations

We want to move data between different frameworks.

- Data is resting in schema $\mathcal{C}$.
- We want to move it in a specific way to schema $\mathcal{D}$.
- We can specify this transformation using functors.

Functorial data migration

We can do all sorts of data transformations using functors.

- Queries, ETL processes, warehousing, schema evolution, etc.

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  - sends nodes to nodes,
  - sends arrows to paths, and
  - respects path equivalence.
- But functors play two roles here:
  - They connect schemas to schemas, $\mathcal{C} \to \mathcal{D}$
  - They connect schemas to data, $\mathcal{D} \to \mathbf{Set}$.
  - Upshot: one can compose and get a functor $\mathcal{C} \to \mathbf{Set}$.

The category of instances

- Given a schema $\mathcal{C}$, the *category of instances* on $\mathcal{C}$ is denoted $\mathcal{C}$-**Inst**.
  - The objects of $\mathcal{C}$-**Inst** are functors (instances) $I: \mathcal{C} \to \mathbf{Set}$.
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- Functors between schemas allow us to move data between them.
  - Given a functor $F: \mathcal{C} \to \mathcal{D}$,
  - There are automatically three data transformation functors

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There's a lot of mathematics ready made for moving data.

Outline

An invitation to engage with us, and solve real-world problems.

The history of AQL

- The mathematical foundations of this story are old.
  - The basic idea was known to mathematicians 60 years ago.
  - More recently we've learned a lot about how to calculate them fast.
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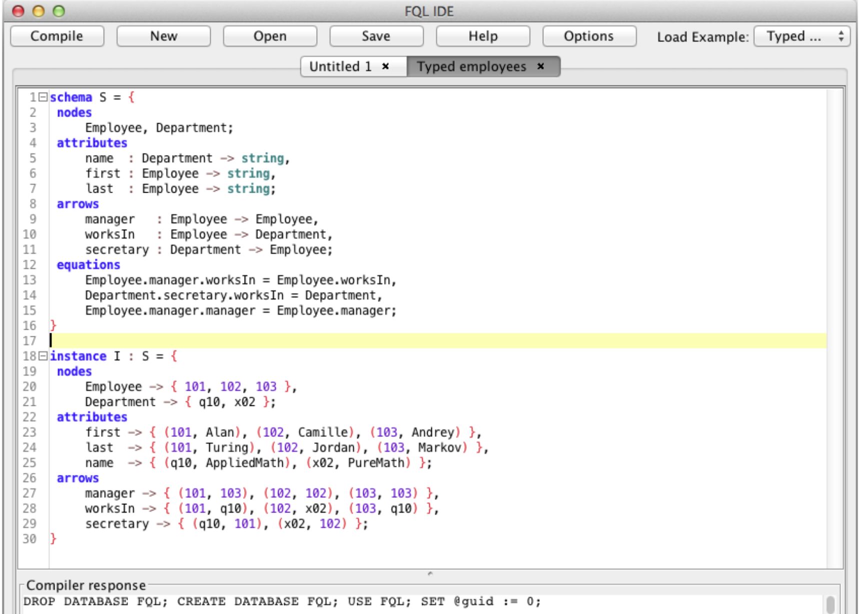
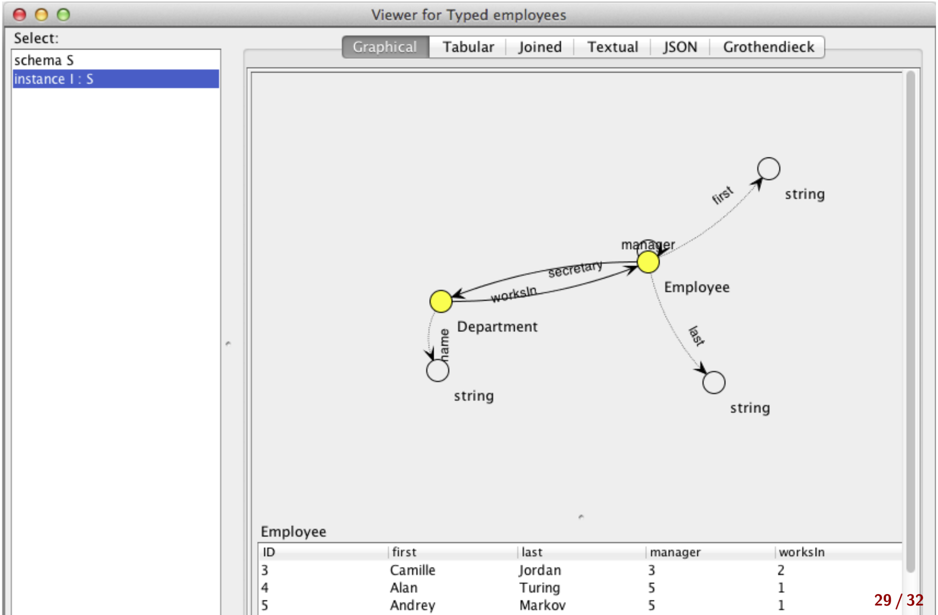
- We received funding from various government agencies.
  - ONR, AFOSR, NIST, NSF.
- A company spun out of MIT in 2015.
  - Categorical Informatics Inc.
  - All MIT IP is open source, all Catinf IP is not.

Screenshot 1



Screenshot 2



ID	first	last	manager	worksIn
3	Camille	Jordan	3	2
4	Alan	Turing	5	1
5	Andrey	Markov	5	1

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- And this database stuff is just one part of category theory.
    - CT has formalized the principles of mathematics, in mathematics.
    - Space, measure, operation, data, symmetry, equivalence, syntax.
    - There is a web of interconnection between all these principles.
- CT been recently highlighted by agencies such as NIST and DARPA.
    - Interdisciplinarity in science and engineering is a big problem.
    - Solving it requires understanding the kinematics of information.

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Once again, CT is *way bigger* than databases.

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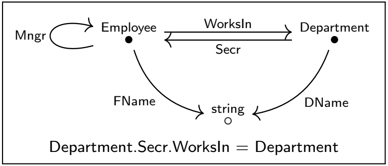

Employee	FName	WorksIn	Mngr
1	Alan	101	2
2	Ruth	101	2
3	Kris	102	3

Department	DName	Secr
101	Sales	1
102	IT	3

String
Alan
IT
⋮

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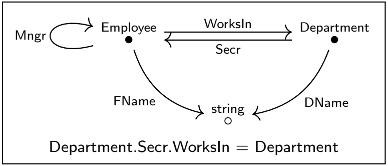

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- Information kinematics—how data moves—is well-modeled by CT.
- Without a good understanding, we waste a lot of time and effort.

Thanks for the invitation to speak!

For more information

Book: *An Invitation to Applied Category Theory: Seven Sketches in Compositionality*. Cambridge University Press, July 2019.
https://arxiv.org/abs/1803.05316

MIT course: Jan 14 – Feb 1, 14:00–15:00, Room 2-143

Company: Categorical Informatics. Website: http://catinf.com