

Petri Banks, quiver representations, and control

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2019 / 01 / 10

Abstract

I'll discuss what the homology of a chain complex has to do with Petri nets, which are popular state-and-transition models used in chemistry and evolutionary game theory. In particular, one can associate an integer matrix to a Petri net, and the homology groups of the corresponding complex correspond to execution-orbits of states and stabilizing executions. I may also discuss some ways that sequences of Petri banks might be useful in control theory.

Petri Banks, quiver representations, and control

I. Introduction

4 mins

A. Petri nets

1. State-and-transition models (Carl Petri 1939)
2. Used in various fields
 - a. Chemical reaction networks
 - b. Evolutionary game theory
 - c. Concurrency theory
3. Can be described by matrices of integers

B. Petri systems

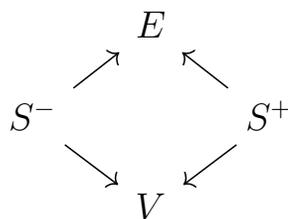
1. A bunch of Petri nets (banks) influencing each other
2. Tight relation to quiver representations
3. Use in control and decision systems

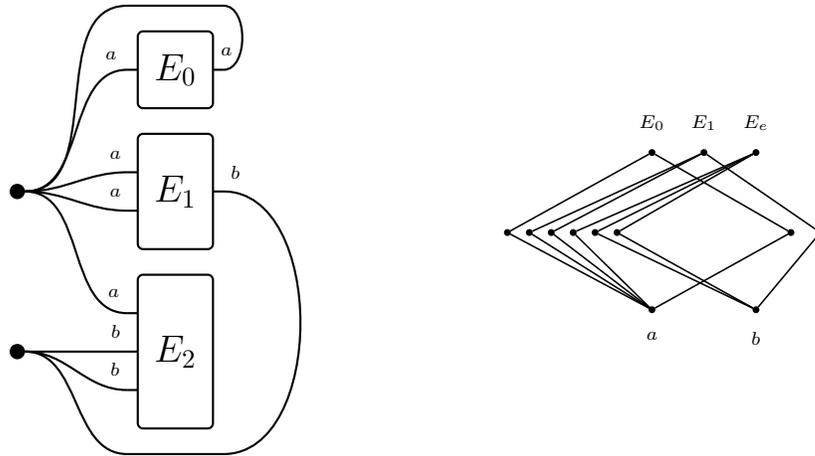
C. Conclude with online learning

II. Petri nets, bipartite graphs, and matrices

9 mins

A. Bipartite graphs





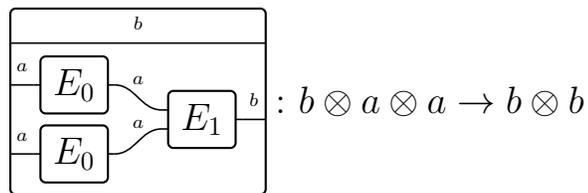
B. Pairs of homo-dimensional \mathbb{N} -matrices

1. Spans $E \leftarrow S \rightarrow V$ (up to isomorphism) $\cong \text{Mat}_{\mathbb{N}}(E, V)$
2. Write s for matrix associated to S .

$$s^- = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \quad s^+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

C. Petri nets (free monoidal cats)

1. Generators: *places* and *transitions*
2. Objects: *states*
3. Morphisms: *executions*



III. Petri banks, matrices, and abelian groups

13 mins

A. Retraction onto \mathbb{Z} -matrices

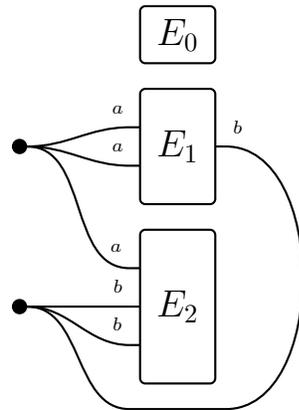
1. Projection: subtraction $s := s^+ - s^-$

$$s = \begin{pmatrix} 0 & -2 & -1 \\ 0 & 1 & -2 \end{pmatrix}$$

2. Inclusion: $s^+ = \max(s, 0)$, $s^- = \max(-s, 0)$

B. A single integer matrix

1. Abelian group presentation
2. Variables: V , equations: E .



$$\begin{aligned} 0 &= 0 \\ 2a &= b \\ a + 2b &= 0 \end{aligned}$$

$$s = \begin{pmatrix} 0 & -2 & -1 \\ 0 & 1 & -2 \end{pmatrix}$$

$$0 \longrightarrow \mathbb{Z}^3 \xrightarrow{s} \mathbb{Z}^2 \longrightarrow \mathbb{Z}_5 \longrightarrow 0$$

C. Cayley groupoid G_s of $s: \mathbb{Z}^E \rightarrow \mathbb{Z}^V$.

1. Category:
 - a. Objects: states \mathbb{Z}^V
 - b. Morphisms: executions $\text{hom}(v, v') = \{e : \mathbb{Z}^E \mid e + v = v'\}$
 - c. Identities: 0
 - d. Composition: +
2. It's a groupoid: inverse of e is $-e$.
3. Monoidal+traced structure

4. $\pi_0 \text{Ob}(G_s)$
 - a. This is execution-orbits of the net
 - b. The quotient group \mathbb{Z}^V / s .

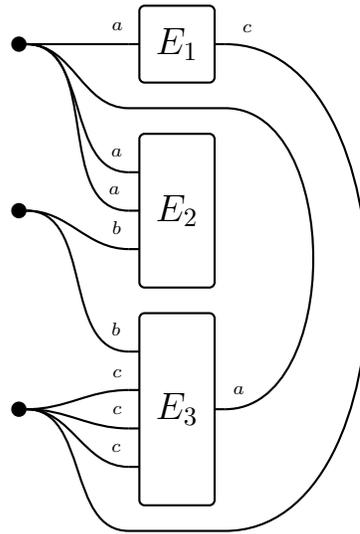
D. Petri banks

1. A traced monoidal category of the form G_s
2. You can owe money
3. Any exchange can be reversed (rolled back)

E. Homology

1. Cokernel: execution-orbits
2. Kernel: stable executions

$$\ker \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ -1 & 0 & 3 \end{pmatrix} = \text{span} \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$$



IV. Decision systems and quiver representations

9 mins

A. Chains

$$\cdots \longrightarrow A_2 \xrightarrow{s_1} A_1 \xrightarrow{s_0} A_0$$

1. As hierarchical decision systems
2. States of A_{i+1} can dictate changes of state in A_i .

3. Complexes seem less interesting than I'd imagined.

B. Loops

$$A \xrightarrow{s} A$$

1. $(n \times n)$ -matrices as dynamical systems
 - a. Actually use $s + I_n$ as dynamical system
 - b. Sends any state v to $sv + v$.
2. Eigenvalues as growth and decay?

C. Arbitrary quiver representations

$$Q = \boxed{Q_1 \begin{array}{c} \xrightarrow{e} \\ \xleftarrow{s} \end{array} Q_0}$$

1. Nice mathematics
 - a. Much is known about quiver reps $B: Q \rightarrow \mathbf{FAb}$
 - b. They form an abelian cat.
 - c. Good graphical logic, etc.
2. B has nice semantics as a system of decision makers
3. B -Behaviors form a sheaf in TTT.
4. Naive (clock-based) dynamical system

$$\sum_{p \in Q_1} B_p: \bigoplus_{i \in Q_0} B_i \longrightarrow \bigoplus_{i \in Q_0} B_i$$

V. Conclusion: learning?

4 mins

- A. Petri banks are just matrices
- B. Matrices can be learned by DNNs
- C. What to train on? Empowerment concept?