Reglog – the game

David I. Spivak
(joint with Brendan Fong)

2019/06/24

Compose :: Conference

Outline

Minority Report

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The following has very little direct relation to the movie Minority Report; it's just an analogy for color.

Minority Report... regular logic style

The 2002 movie *Minority report* showed detective Tom Cruise playing seamlessly with logic.

- A computer database held relevant information.
- Cruise could pull it up and manipulate it to solve crimes.

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Let's imagine our own version of a detective scenario.

Brought to you by... regular logic – the game!

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- You're adding to and narrowing down your set of suspects.
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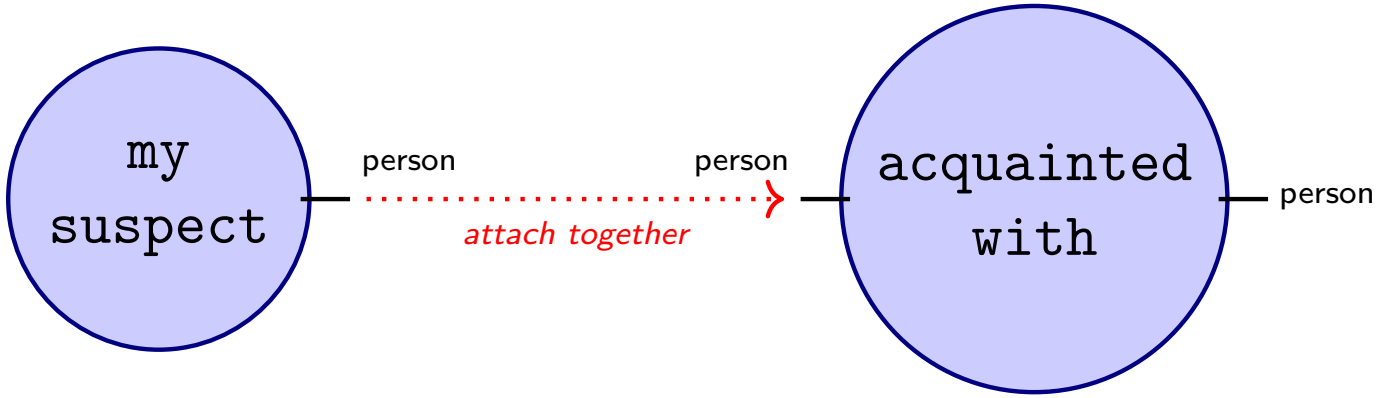
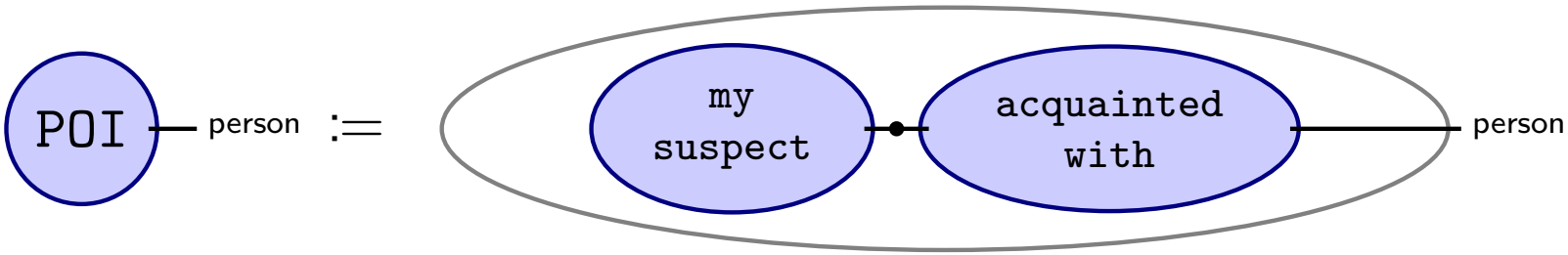


- and define `POI` (person of interest) as the result:



Adding beliefs

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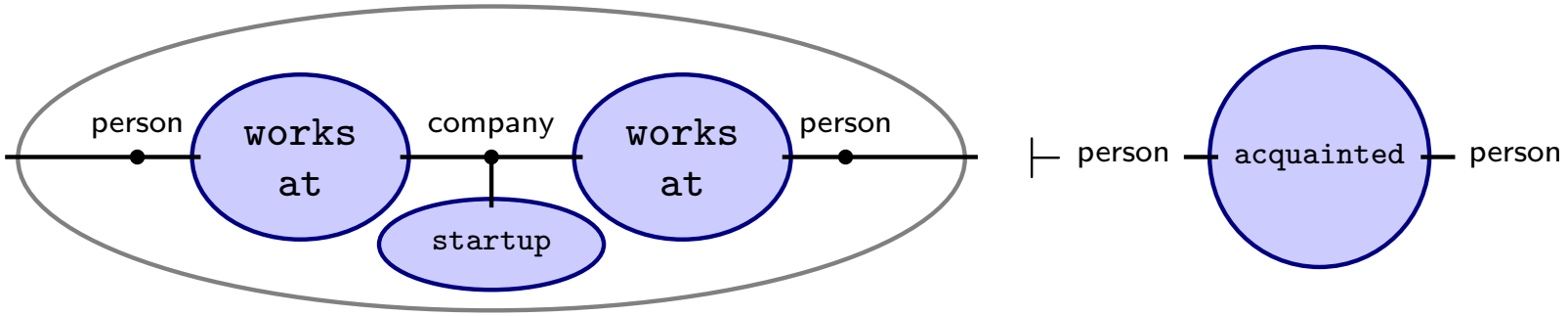

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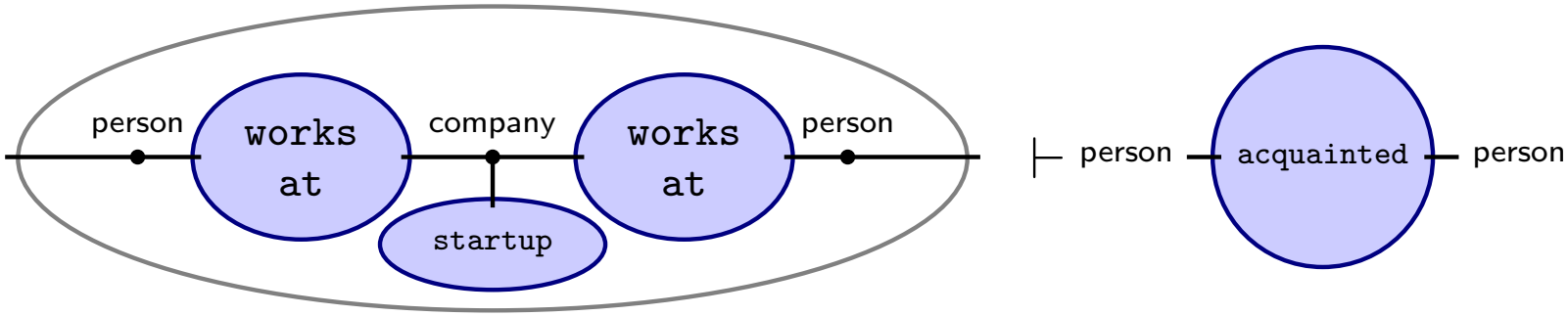

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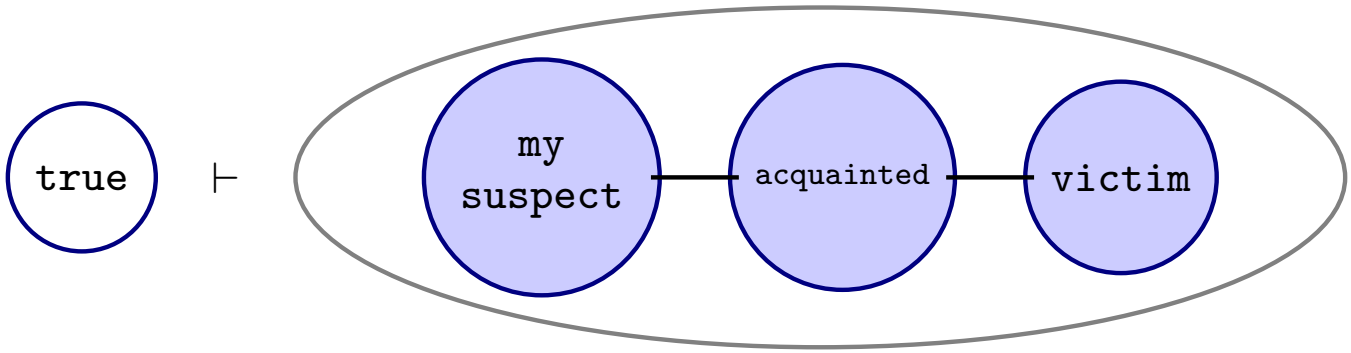

Belief: "In any case, my suspect is acquainted with the victim."

Compositionality

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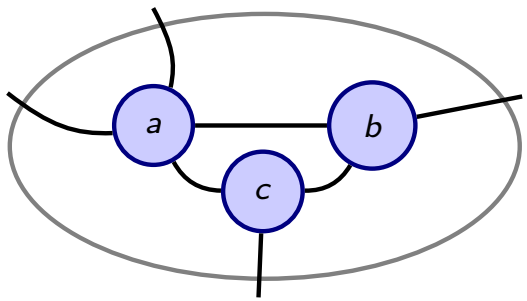

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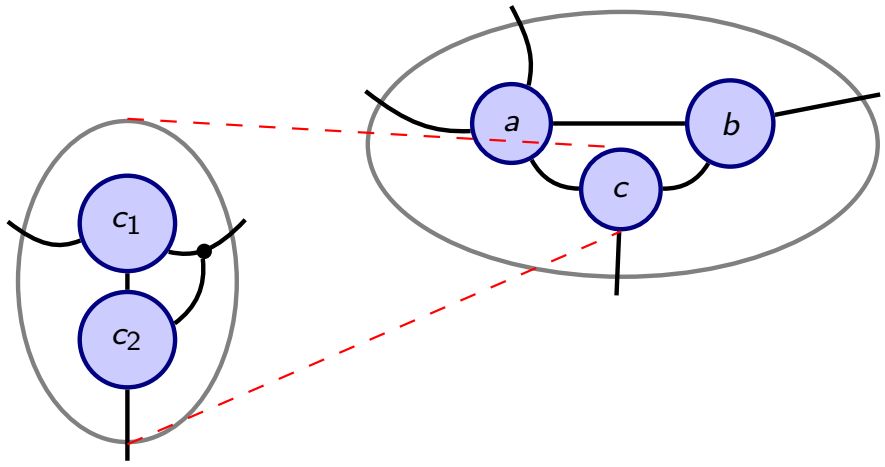

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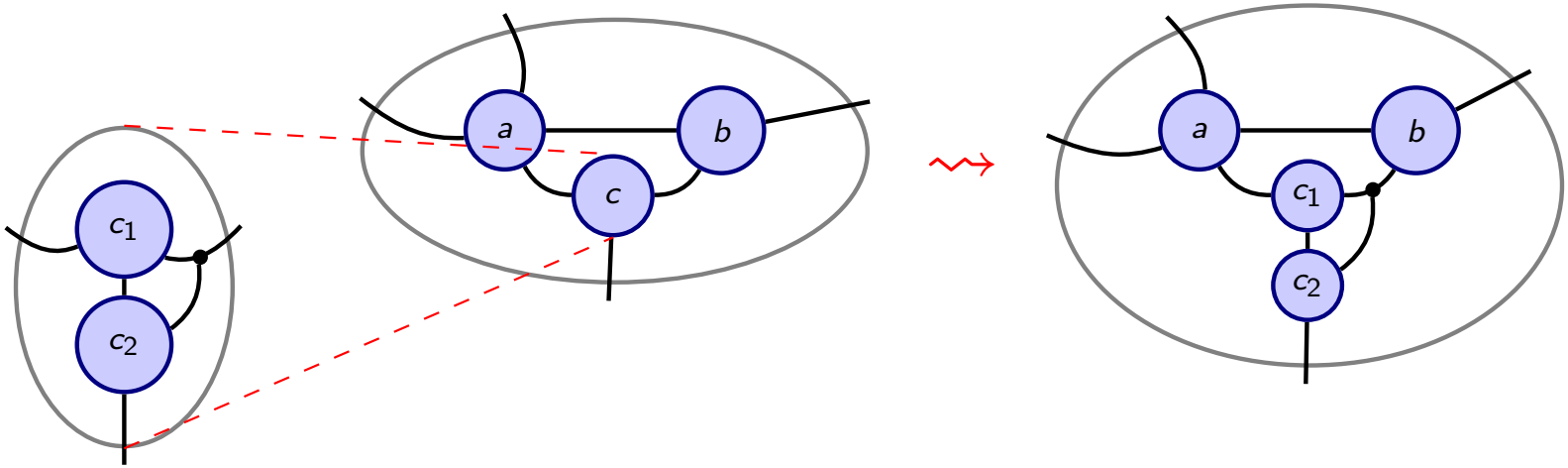

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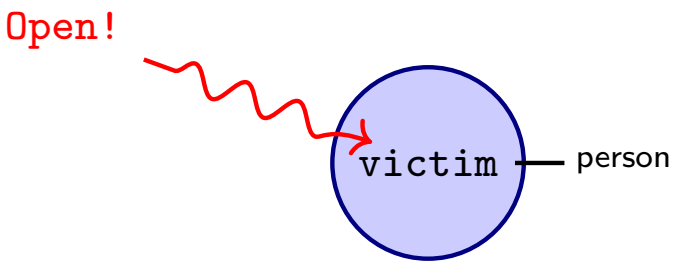

victim
person
P105
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Persons are internal identifiers; we want to see facts about the victims.

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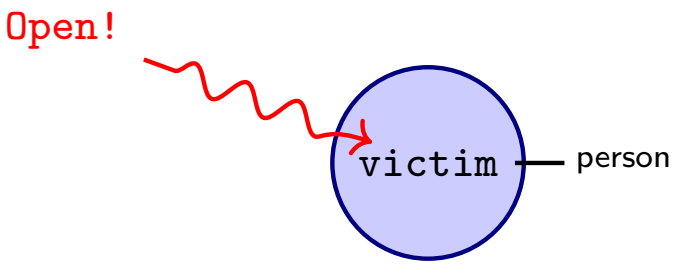

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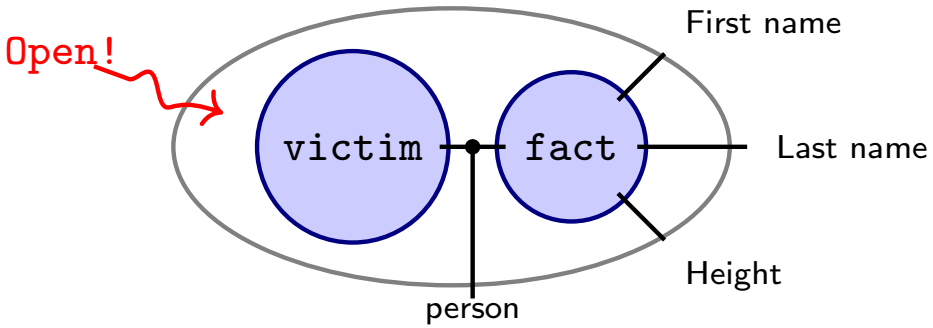

victim fact			
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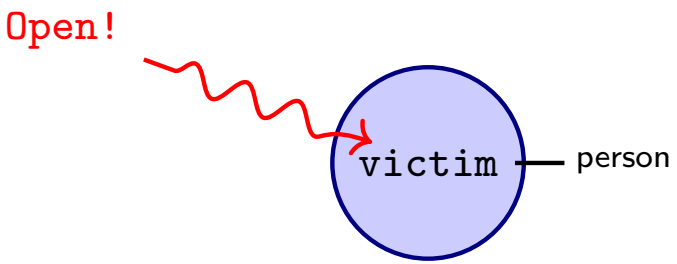

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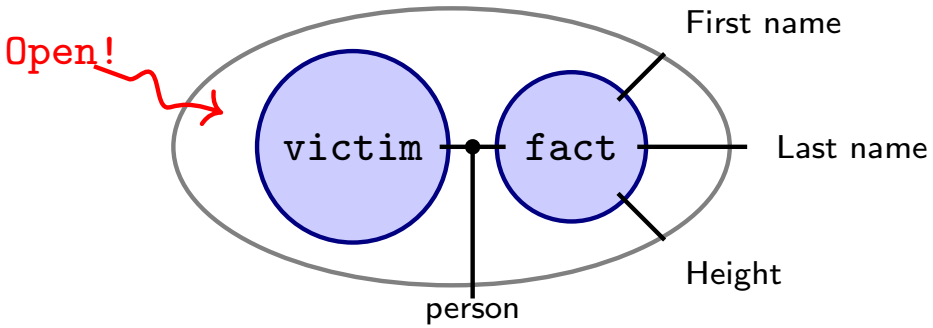

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Some knowledge may be missing or otherwise imperfect.

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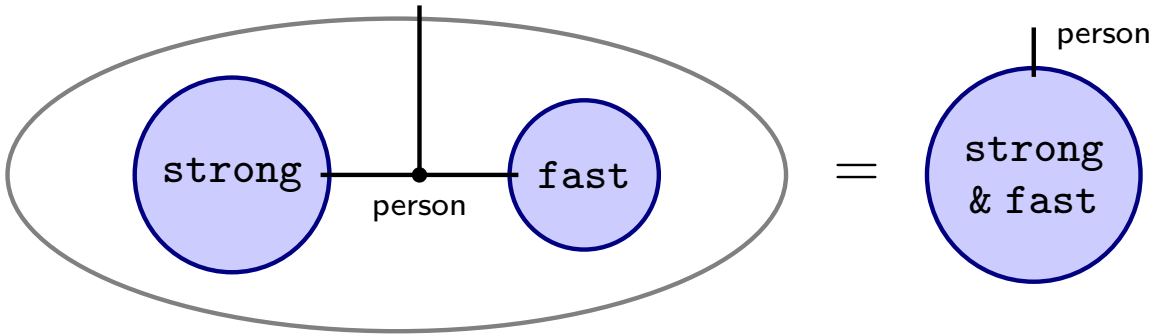

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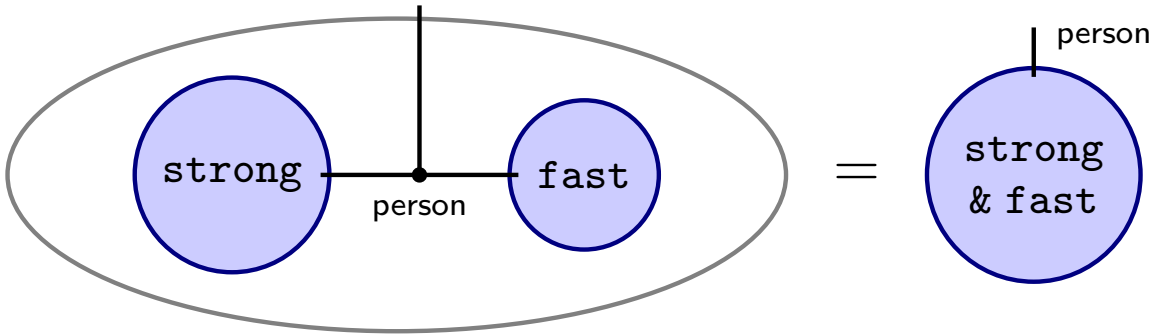

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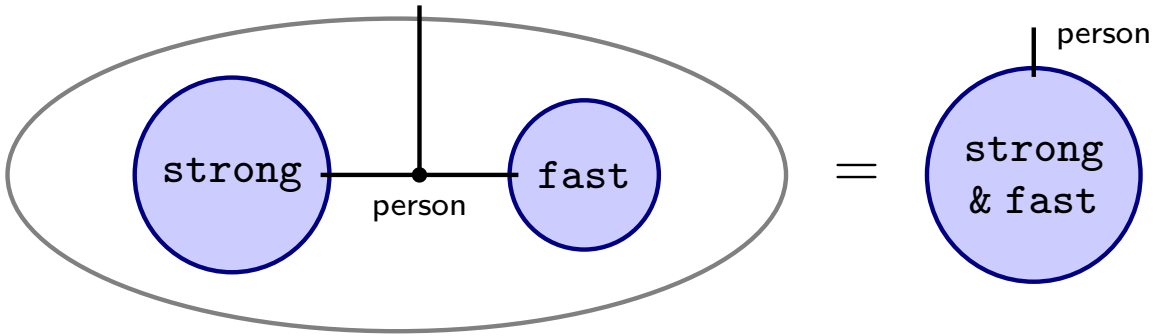

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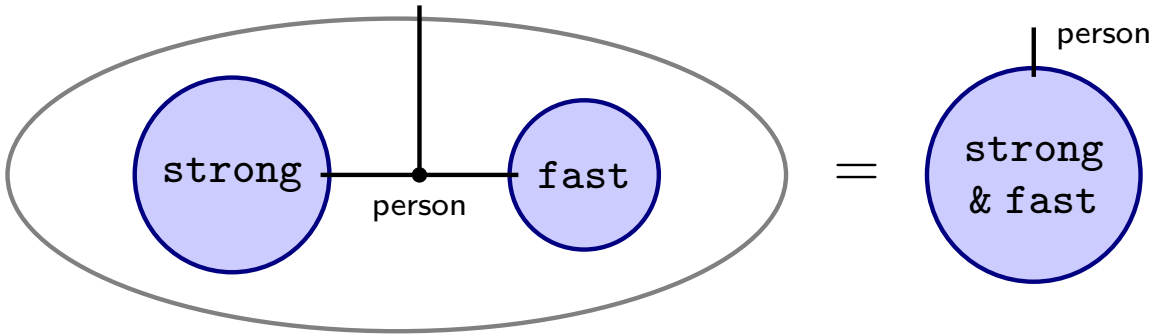

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Reasoning: regular "old" logic, with a shiny new math-specified GUI.

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The chase minimally "repairs" $I \to I'$, with I' conforming to axioms.

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Can we make this real?

Plan for rest of talk

"Detective" is not the only game in reglog – the game.

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- But it can also be used to specify DSLs.
- E.g. Tom Ellis' Haskell package *Opaleye* is based on similar work.
- Today's talk is a generalization of that story.

Outline

Regular logic and regular categories

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- Regular logic is the internal logic of regular categories.
- Examples of regular categories:
  - Set, and more generally any topos;
  - $\mathrm{Set}^{\mathrm{op}}$, opposite of any topos, also $\mathrm{TopSp}^{\mathrm{op}}$;
  - The category of models of any Lawvere theory (Groups, Rings, ...);
  - The slice (also the coslice) of any regular category over any object;
  - Exponential ideal: if $\mathcal{R}$ regular and $\mathcal{C}$ a category, then $\mathcal{R}^{\mathcal{C}}$ is regular.

Regular categories have well-behaved relations

Regular categories $\mathcal{R}$ are those with a good *2-category of relations*.

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Regular categories have enough structure to do regular logic.

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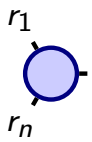
  - Have a shell 

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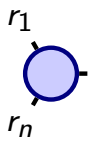
  - Have a shell 

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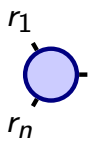
  - Have a shell 

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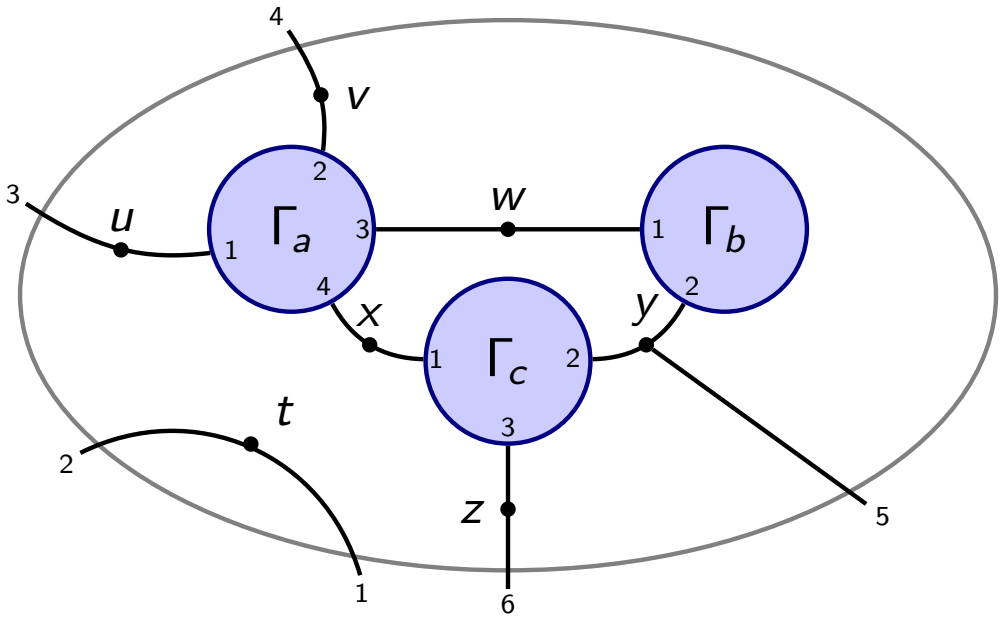
Let's discuss wiring diagrams.

Mathematical specification of wiring diagrams

Mathematically, what is a wiring diagram $(\Gamma_a, \Gamma_b, \Gamma_c) \to \Gamma'$?



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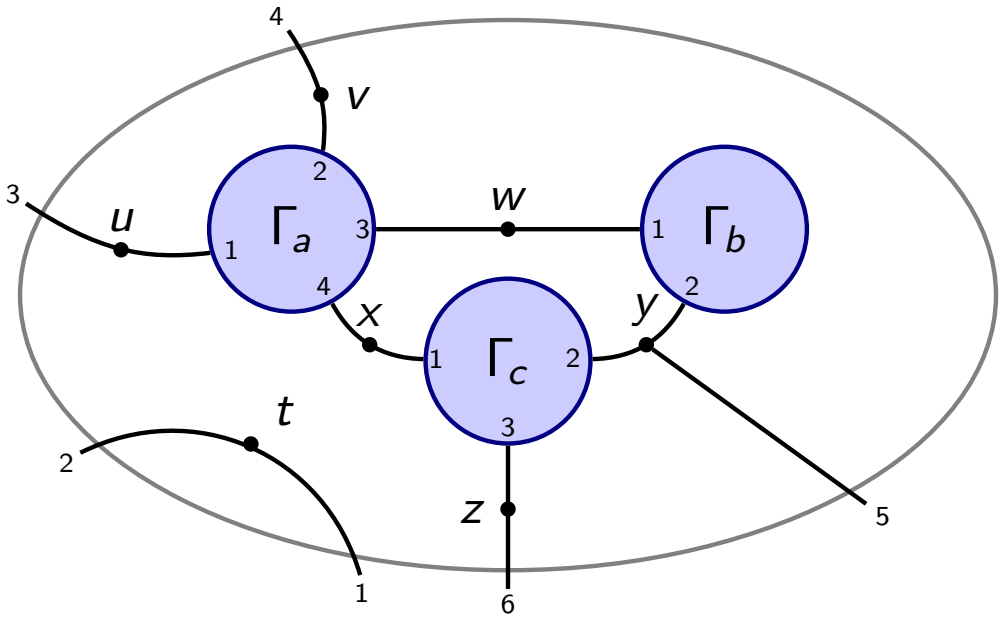
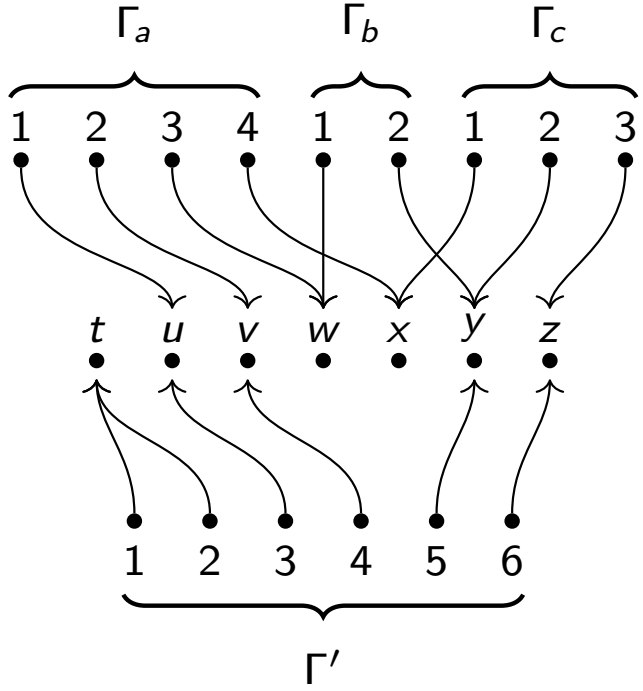




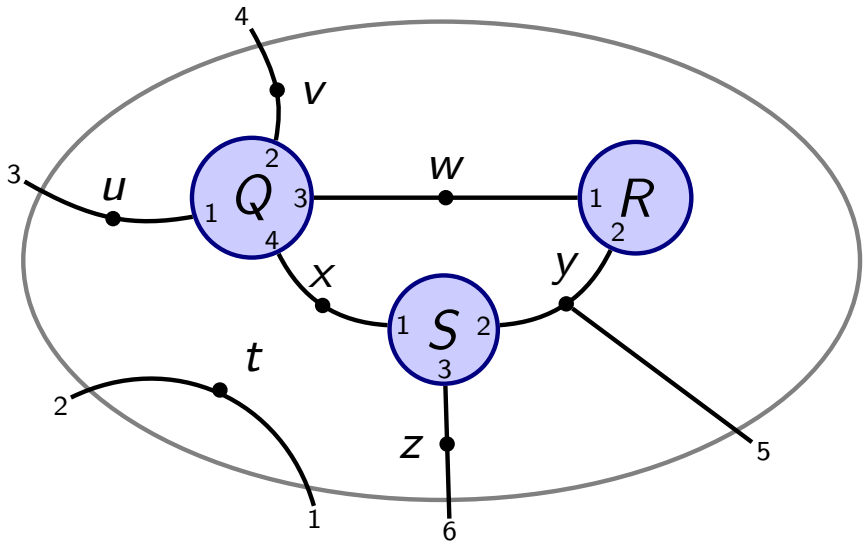
It is a morphism $(\Gamma_a, \Gamma_b, \Gamma_c) \to \Gamma'$ in the operad of cospans.

- That is, it's a *cospan*: a pair of functions with common codomain

$$\Gamma_a \sqcup \Gamma_b \sqcup \Gamma_c \to \{t, \ldots, z\} \leftarrow \Gamma'$$

Wiring diagrams as logical expressions

We can convert a wiring diagram like this into a logical expression:



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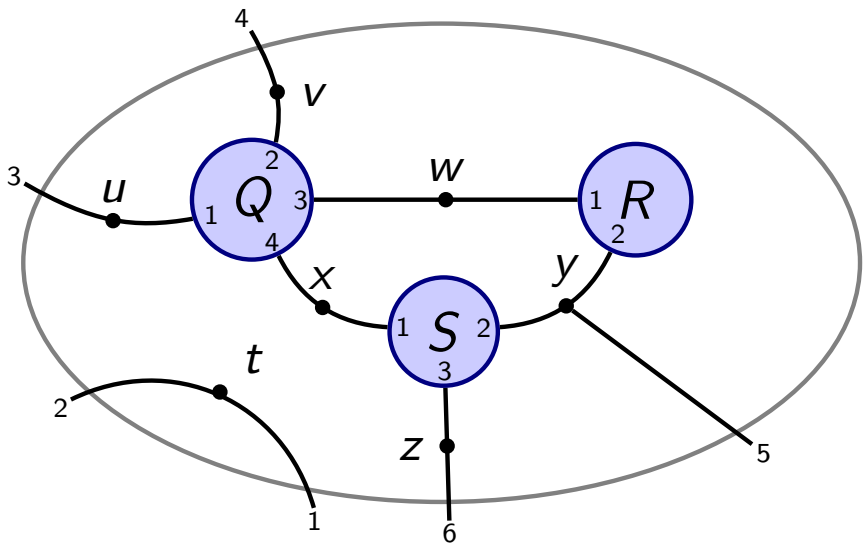
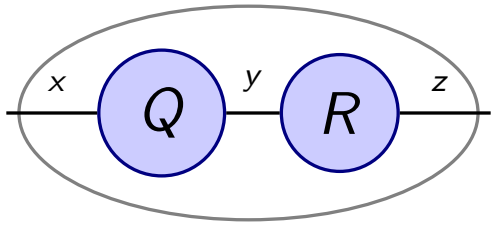


- Write type of exterior shell Γ', naming each port by a distinct variable.
- Write quantifier $\exists(x : X)$ for each unexposed wire of type X.
- AND together internal cells, with established var. names from above.
- Equate variables for exposed ports that are connected.

$$O(t_1,t_2,u_3,v_4,y_5,z_6) := \exists(w{:}W, x{:}X).\, Q(u_3,v_4,w,x) \wedge R(w,y_5) \wedge S(x,y_5,z_6) \wedge (t_1 = t_2)$$

Simpler example



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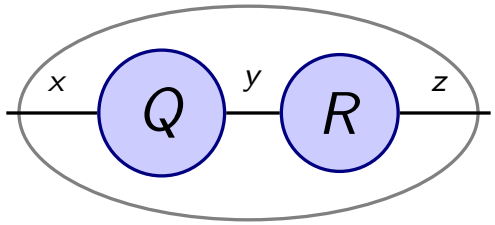

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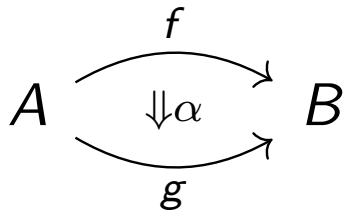

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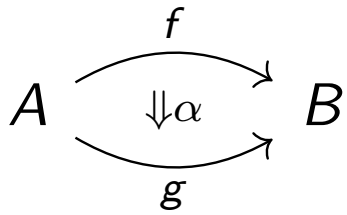

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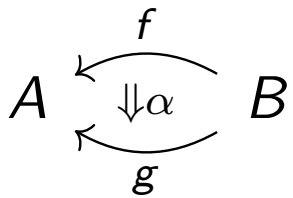

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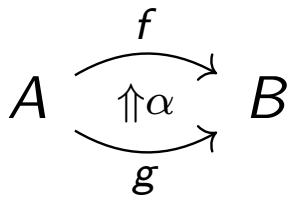

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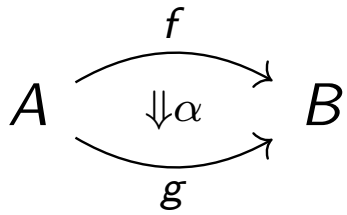

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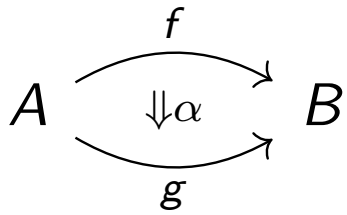

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- These operate on morphisms too (think $f_1 \odot f_2 \colon A_1 \odot A_2 \to B_1 \odot B_2$).
- There's also a monoidal unit (think unit or void).

Formal specification of graphical calculi I

Our "minority report" detective GUI can be understood as follows.

- Fix a set T (each $t \in \mathsf{T}$ is a string label: person, height, etc.).
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- Detective will be a certain kind of 2-functor $P\colon \mathbb{C}\mathrm{ospan}^{\mathrm{co}}_{\mathsf{T}} \to \mathbb{P}\mathrm{oset}$.
  - To each shell $\Gamma \in \mathbb{C}\mathrm{ospan}^{\mathrm{co}}_{\mathsf{T}}$, a poset $P(\Gamma)$.
  - We denote the order in $P(\Gamma)$ using the logical *entailment* symbol $\vdash$.

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Definition

An *regular calculus* is a lax monoidal 2-functor

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Theorem

There is an adjunction

$$\mathsf{RegCalc} \underset{\mathsf{prd}}{\overset{\mathsf{syn}}{\rightleftarrows}} \mathsf{RegCat} \ ,$$

such that for any regular category $\mathcal{R}$, the counit $\mathsf{syn} \circ \mathsf{prd}(\mathcal{R}) \to \mathcal{R}$ *is an equivalence of categories.*

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- Each game developer must supply their own P.
  - To each shell Γ, specify elements of $P(\Gamma)$, drawn as cells (fillers).
  - Specify definition of $\varphi_1 \leq \varphi_2$, drawn perhaps as $\varphi_1 \vdash \varphi_2$.
  - To each wiring diagram w, supply monotonic function $P(w)\colon P(\Gamma_1) \times \cdots \times P(\Gamma_n) \to P(\Gamma')$.
  - Ensure P preserves composition, identity, and dot-breaking.

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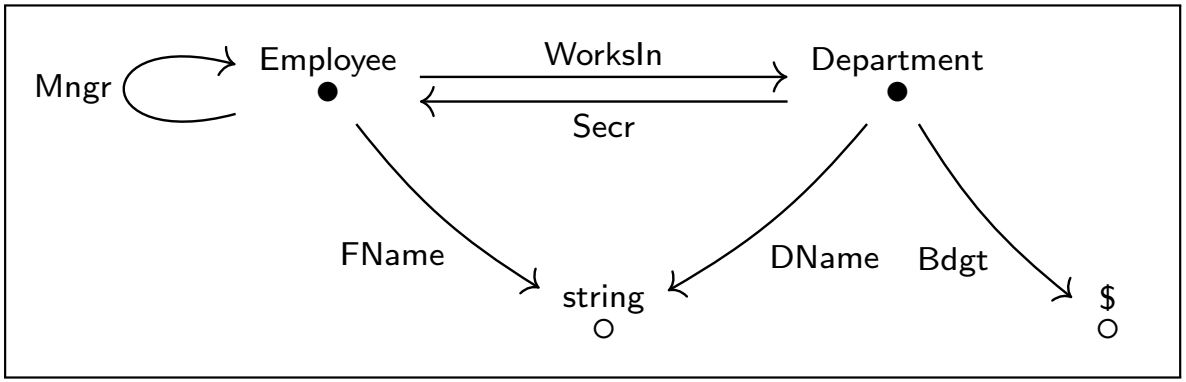

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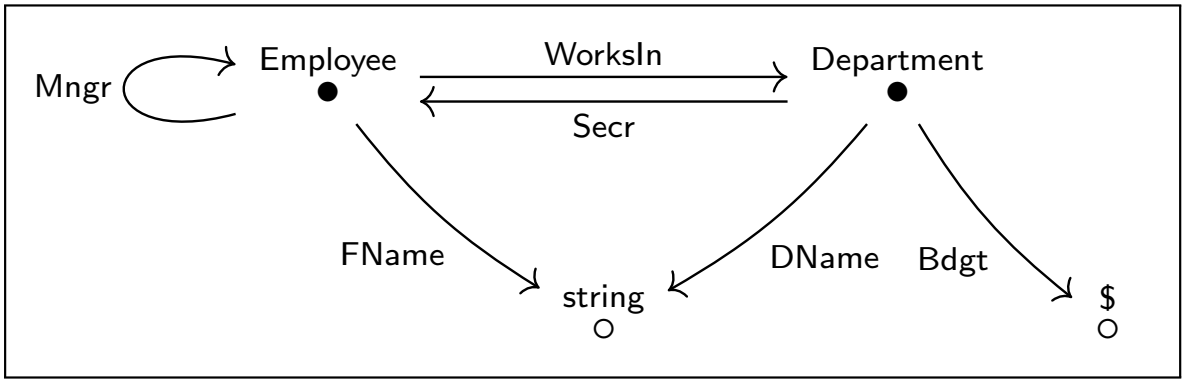

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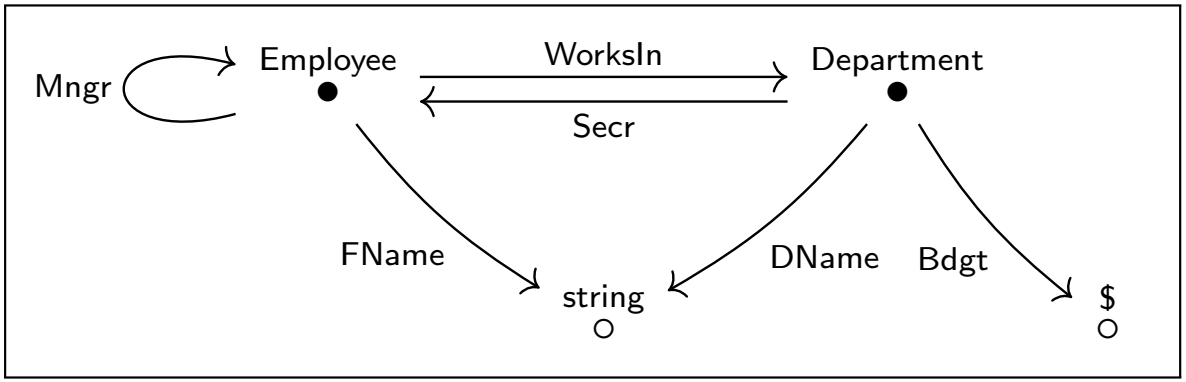

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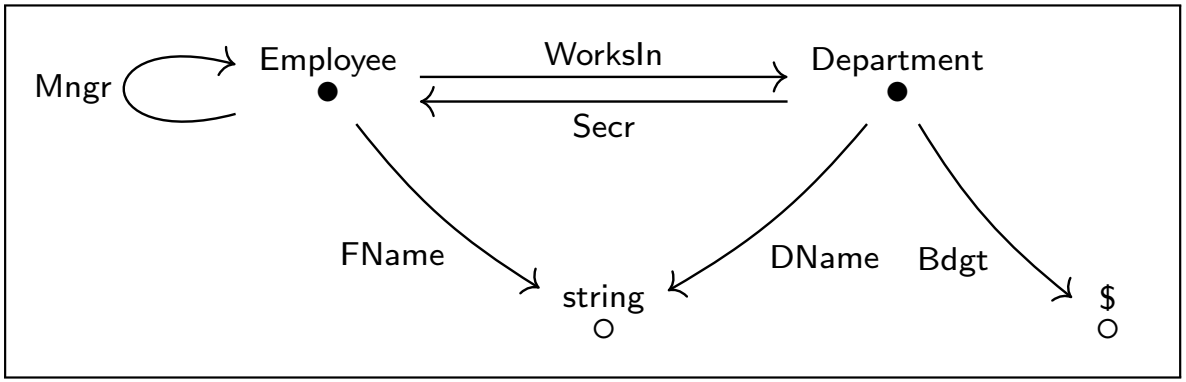

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- Each regular sequent $\phi(\vec{x}) \vdash \psi(\vec{x})$ is called an embedded dependency.
- Chase these EDs to force axioms to hold with minimal other assumptions.

Outline

Common game platform

"Reglog – the game" is actually a bunch of games.

- In common: same GUI and same sort of interaction:
  - Create new shells, attach shells, compose wiring diagrams
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- Some examples:
  - Where are my keys?
  - Smart witter
  - Partitions puzzle
  - Boolean circuits
  - Solve equations
  - etc.

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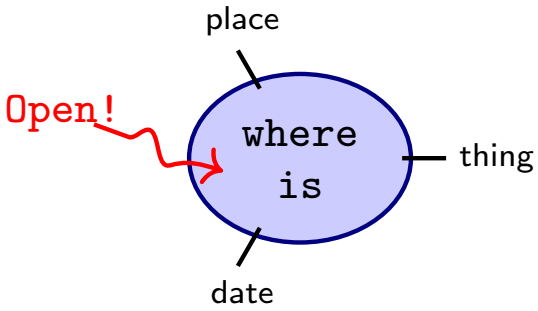

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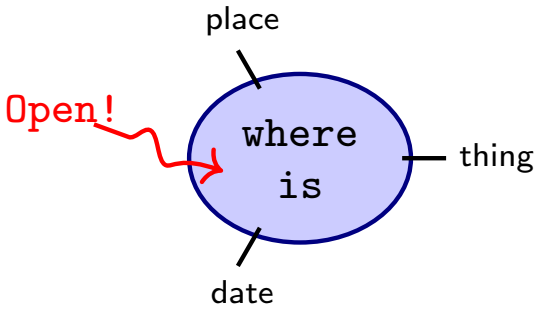

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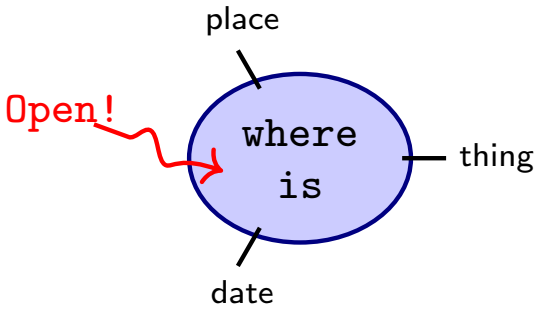

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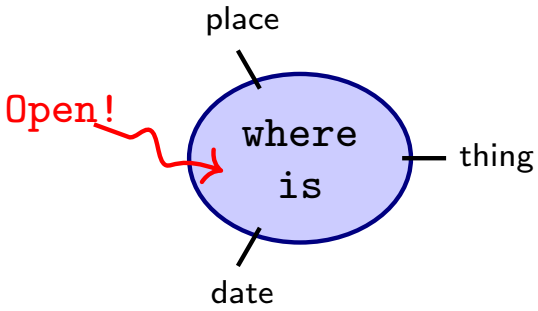

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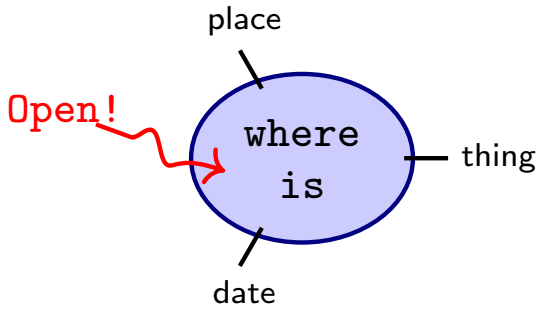

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  - There is an internet-published *doctor* cell doctor — specialty .
  - Connect specialty — cardiologist to doctor's speciality port.
  - Connect doctor's availability and insurance ports to your own.
  - Output doctor's name and phone number.

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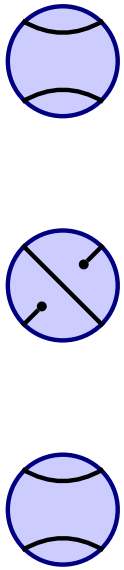
    - Out of all configurations, some are very popular, i.e. reused.
  - Same idea for human concepts; find reusable ideas (memes).

Game: Partitions puzzle

Put pieces:



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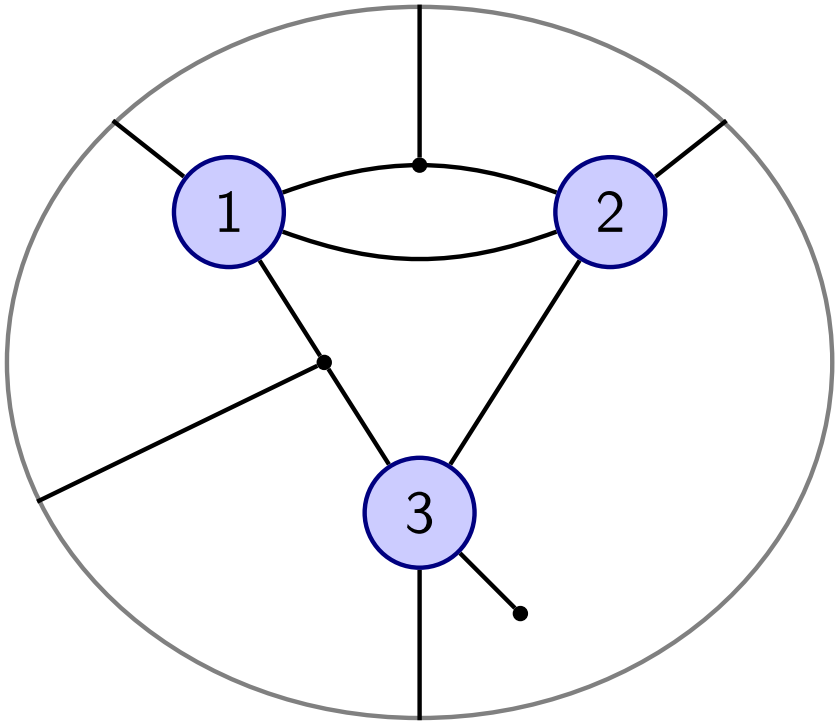

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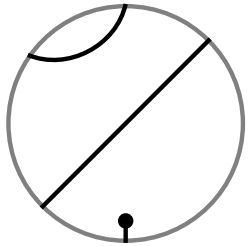

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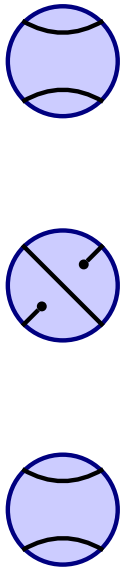

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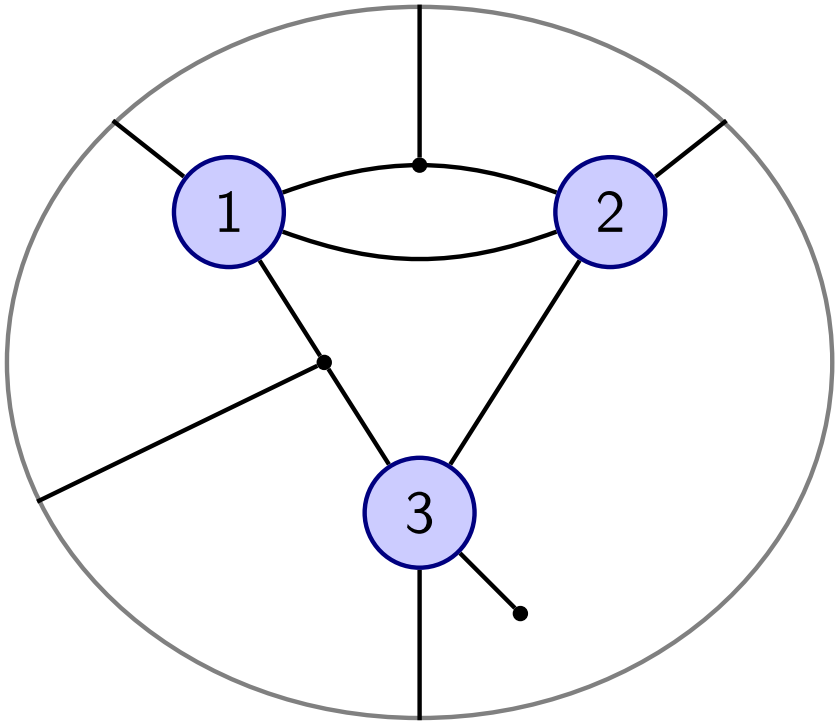

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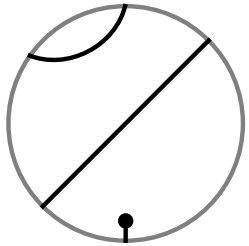
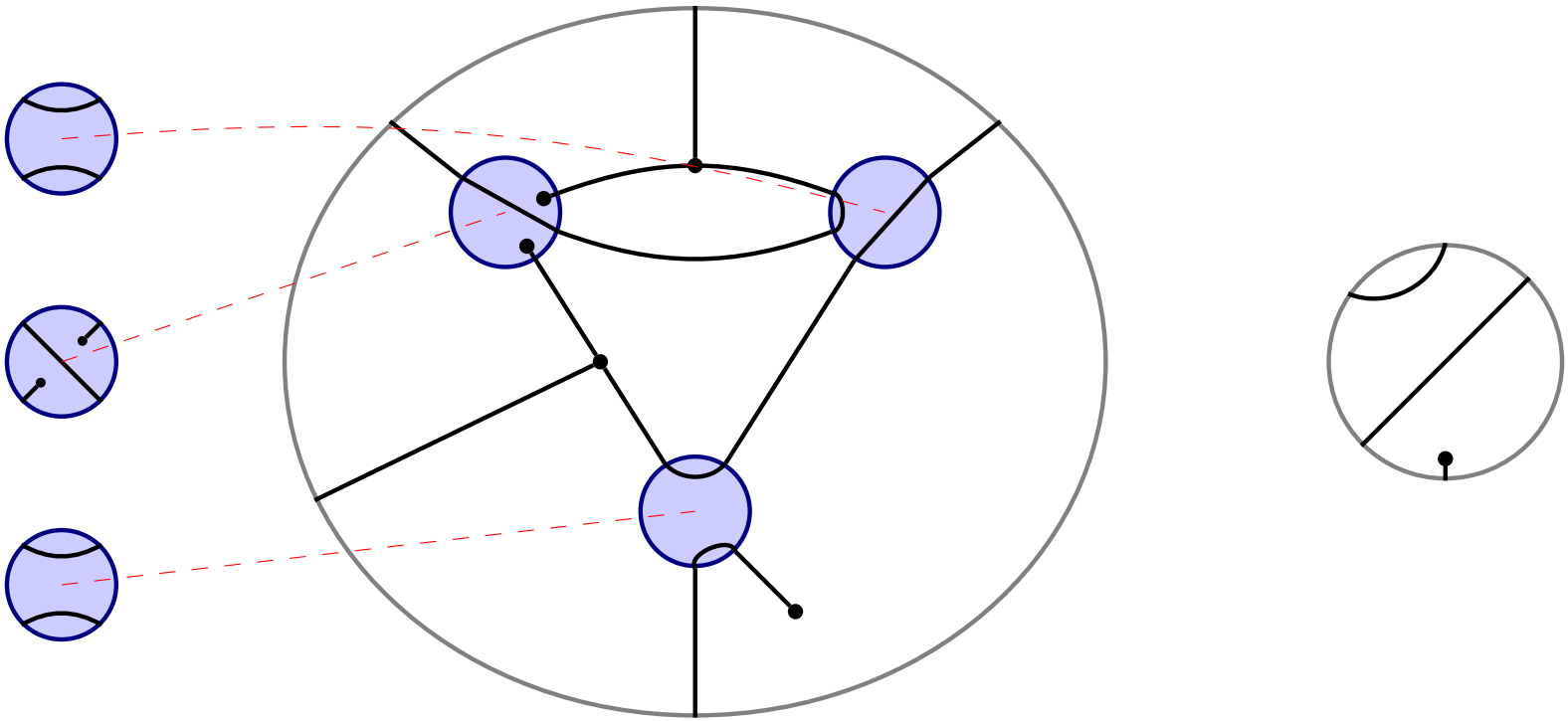


I'll give you a minute to solve it.

Game: partitions puzzle (solution)



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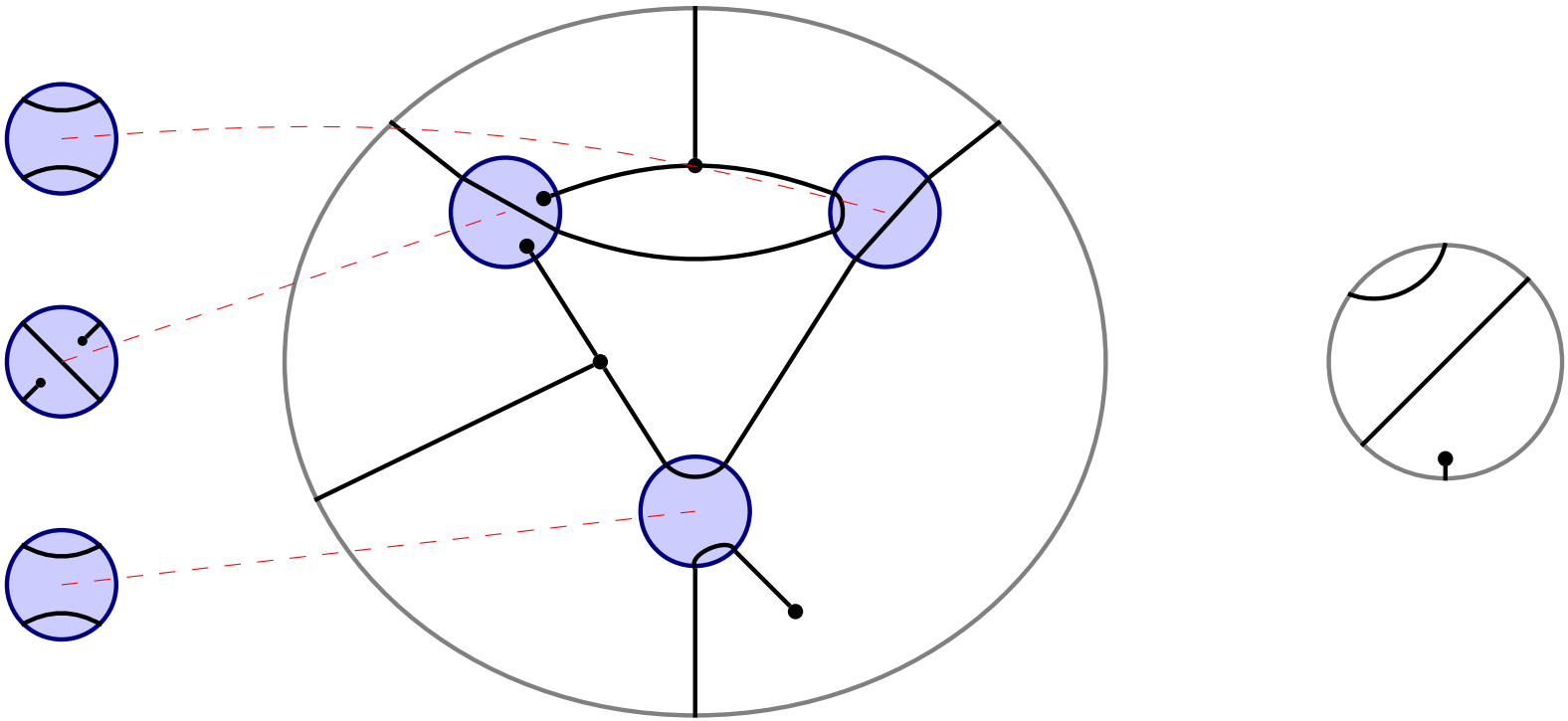

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- Since wires are unlabeled, we see that $\mathsf{T} = ()$ is unit.
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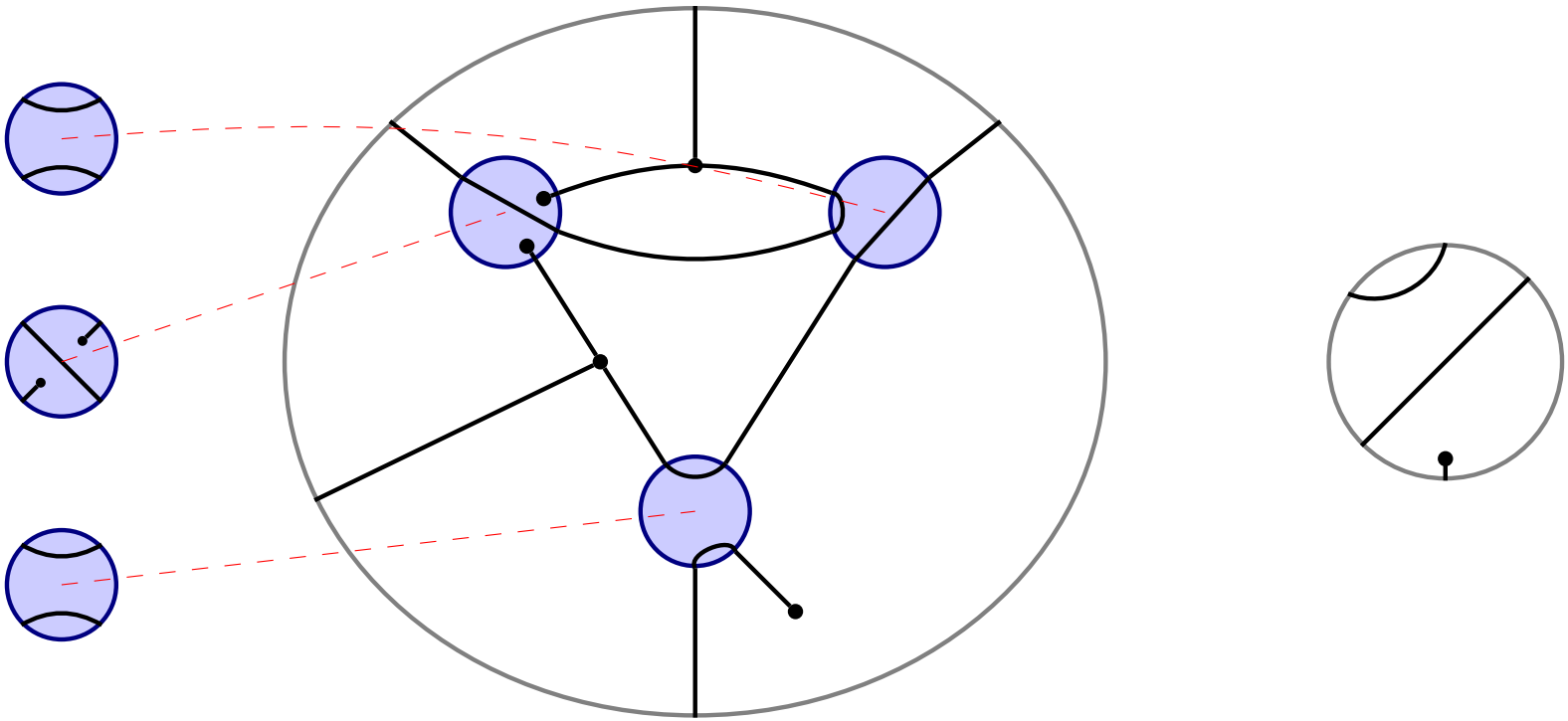

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Boolean circuits are special cases of boolean relations.

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  - A boolean relation is a subset of $\mathbb{B}^n = \{\texttt{true}, \texttt{false}\}^n$ for some n.
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  - Consider also relations, like $\leq$.

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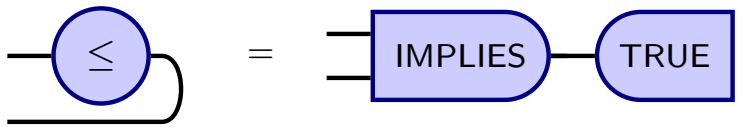

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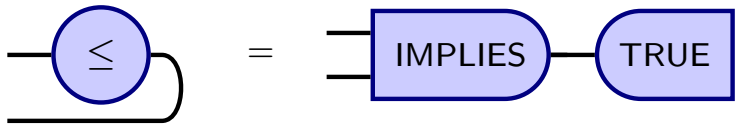

  - Example: every boolean circuit can be built out of NAND gates.

Q: Are there kids who might have fun playing with something like this?

Game: Solving equations

Consider an arbitrary system of equations having the following form:

$$f_1(\mathbf{t}, u, \mathbf{v}) = 0$$
$$f_2(\mathbf{v}, w, x) = 0$$
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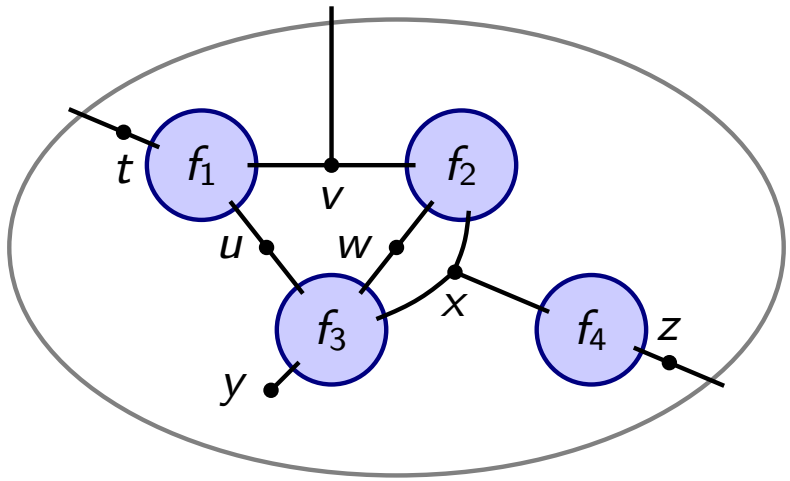

Said another way, we want $\{(t, v, z) \mid \exists u, w, x, y : f_1 = f_2 = f_3 = f_4 = 0\}$.

Systems of equations via pixel arrays

Consider just two equations $f(x, w) = 0$ and $g(w, y) = 0$.

- Plot each in its own bounding box, say in the range $[-1.5, 1.5]$.
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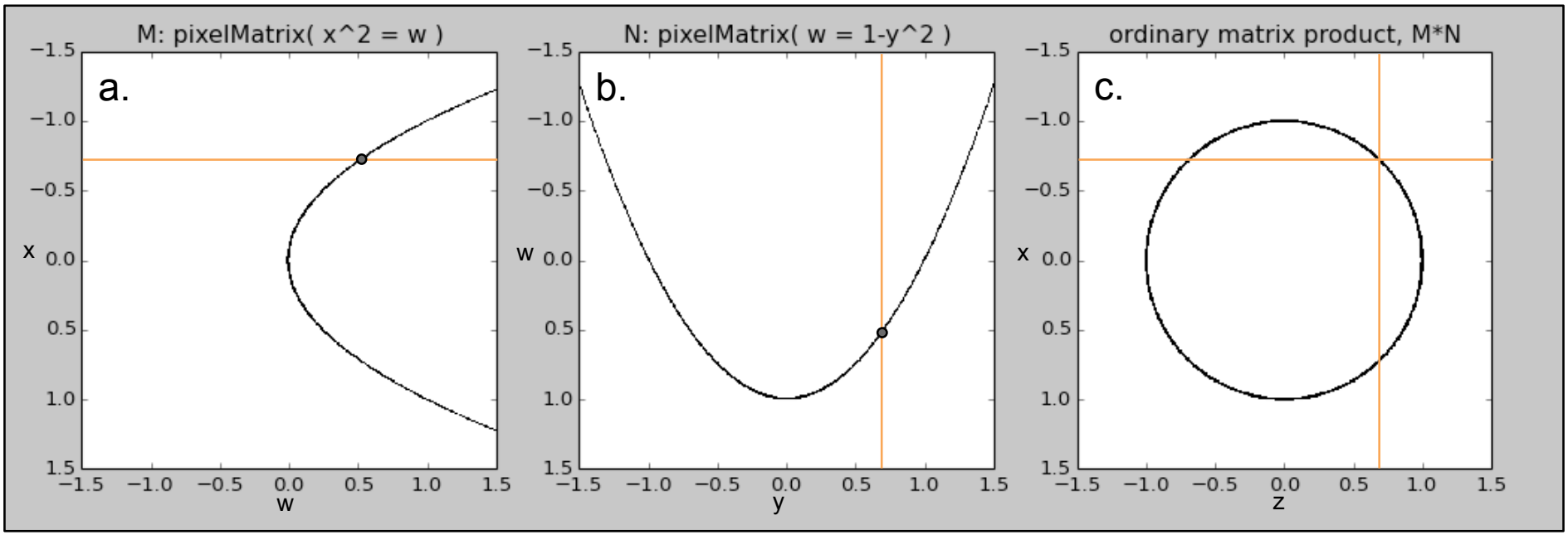
- For example, plot equations $x^2 = w$ and $w = 1 - y^2$, and multiply.



A more complex example

The following eq's are not differentiable, nor even defined everywhere.

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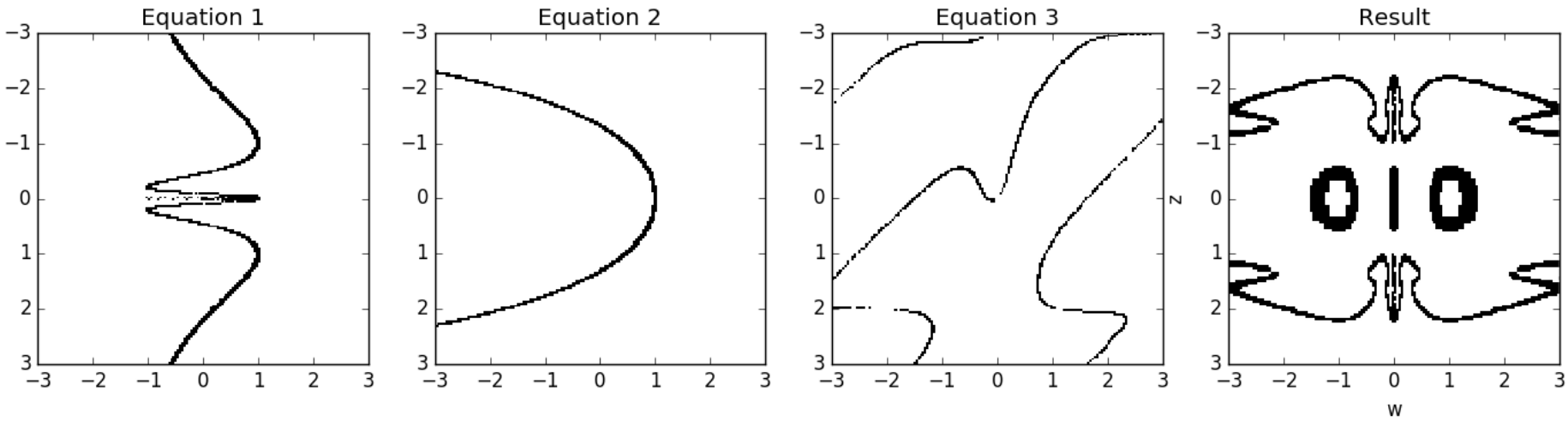

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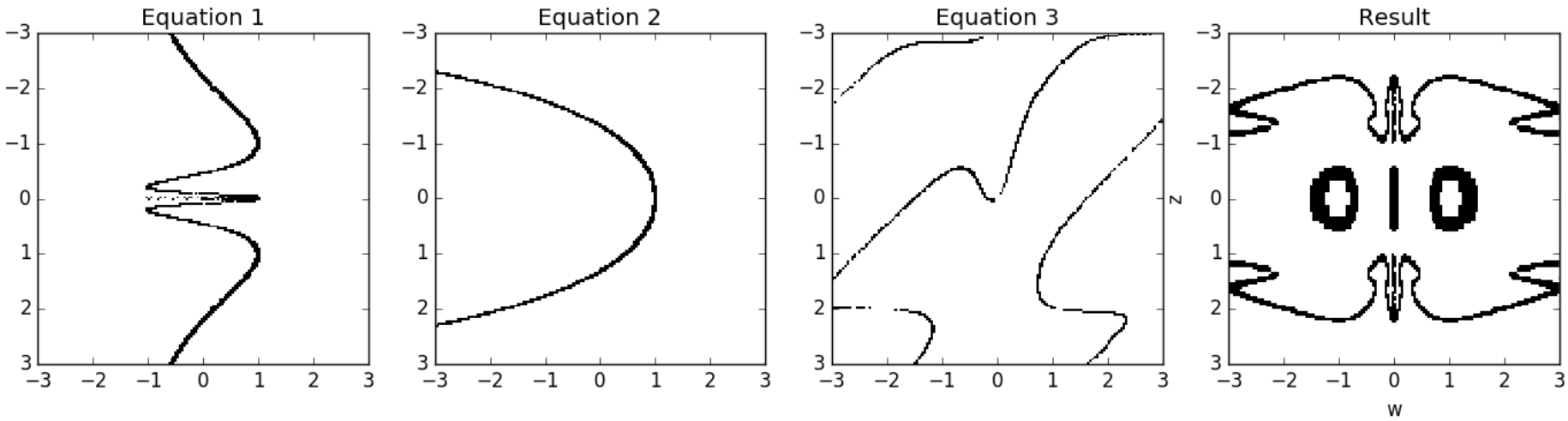

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Game: theory of a group, ring, etc.

Monoids, groups, rings, modules: each has an associated algebraic theory.

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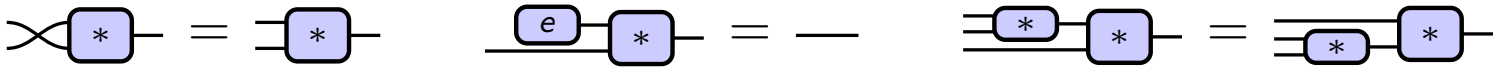

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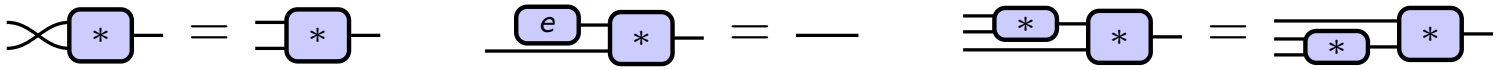

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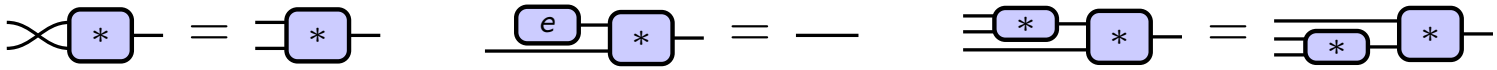

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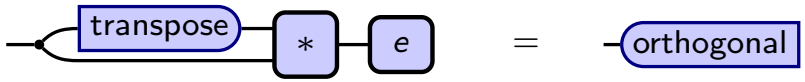
  - E.g. add orthogonal and transpose and axiomatize them.



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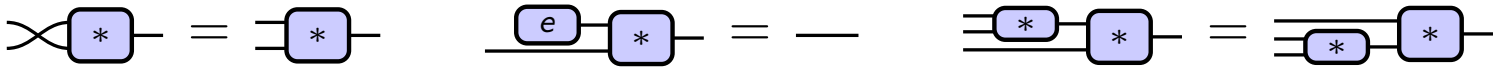

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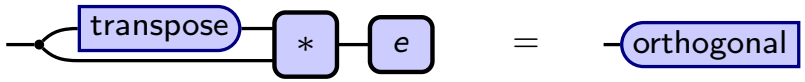
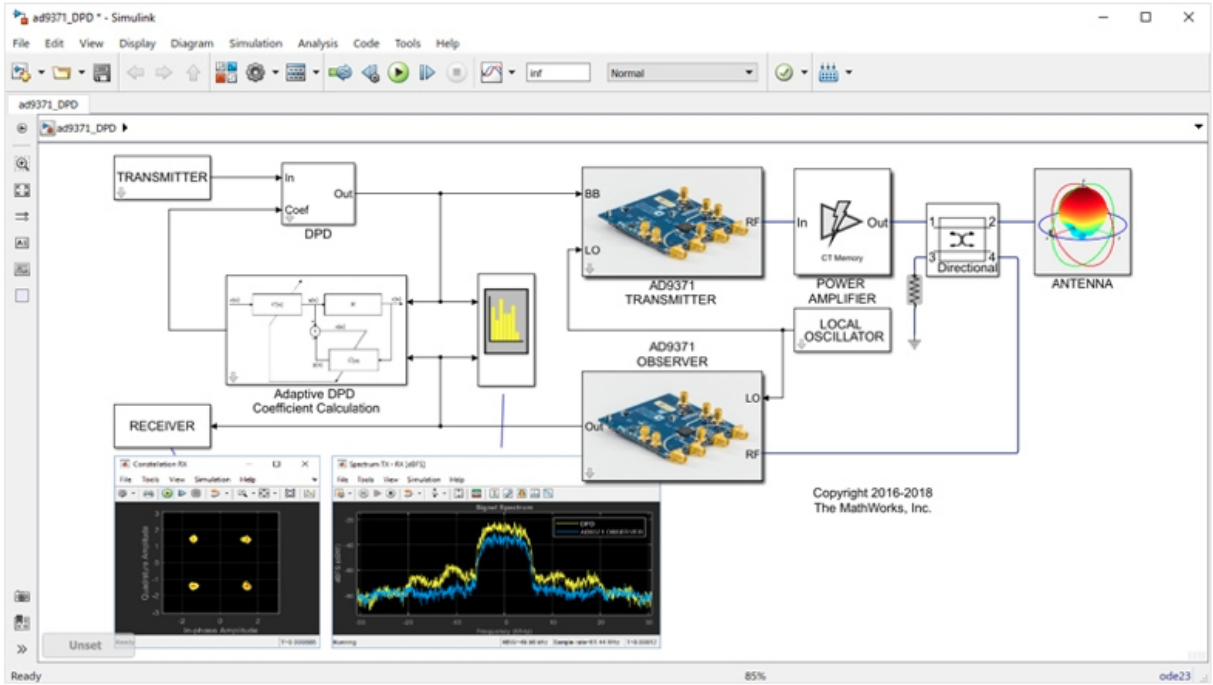


Like "fold-it" for protein folding, players can help w/o understanding.

Game: Simulink

- Simulink (makers of Matlab): model and simulate dynamic systems.



- Connect up smaller dynamic systems.

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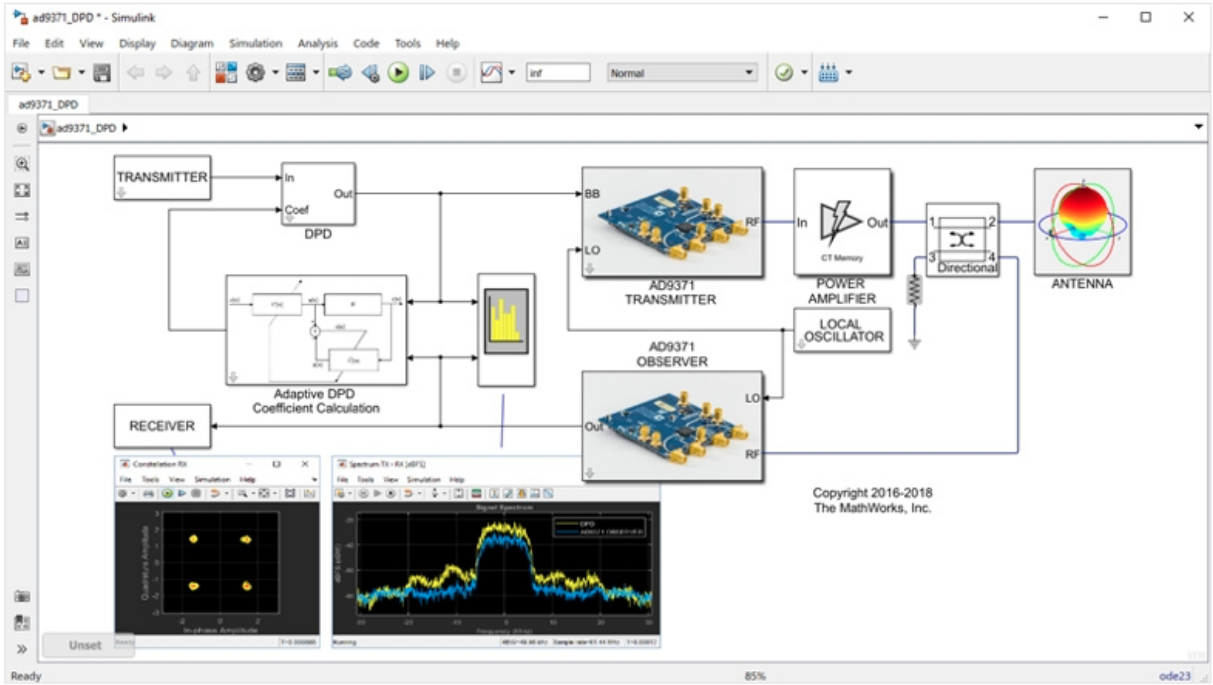
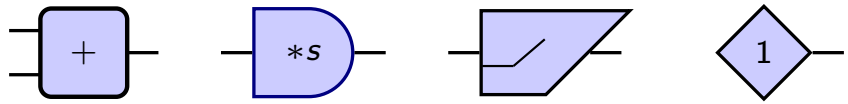


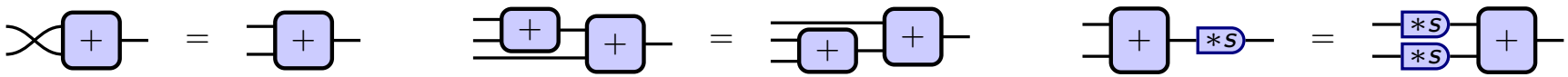
- Connect up smaller dynamic systems.
- Each can be understood as a relation in the temporal topos (TTT).
- Use reglog – the game as a shell for Simulink.

Game: neural relations

Neural networks are formed out of addition, scalar mult, ReLU, and 1.



These satisfy various relations, e.g.



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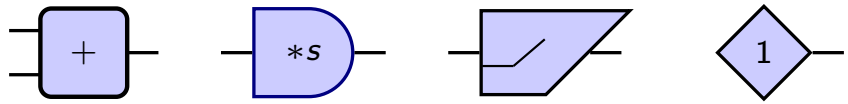

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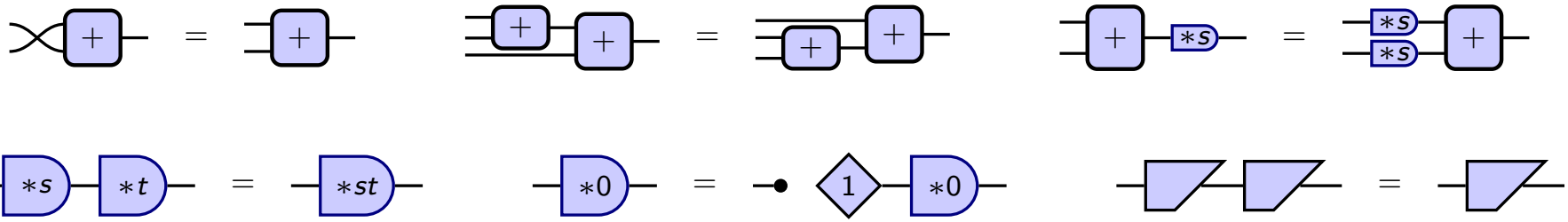

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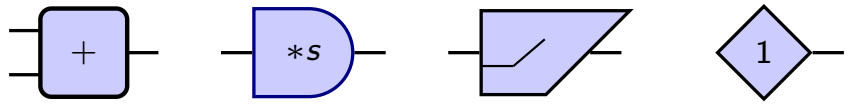

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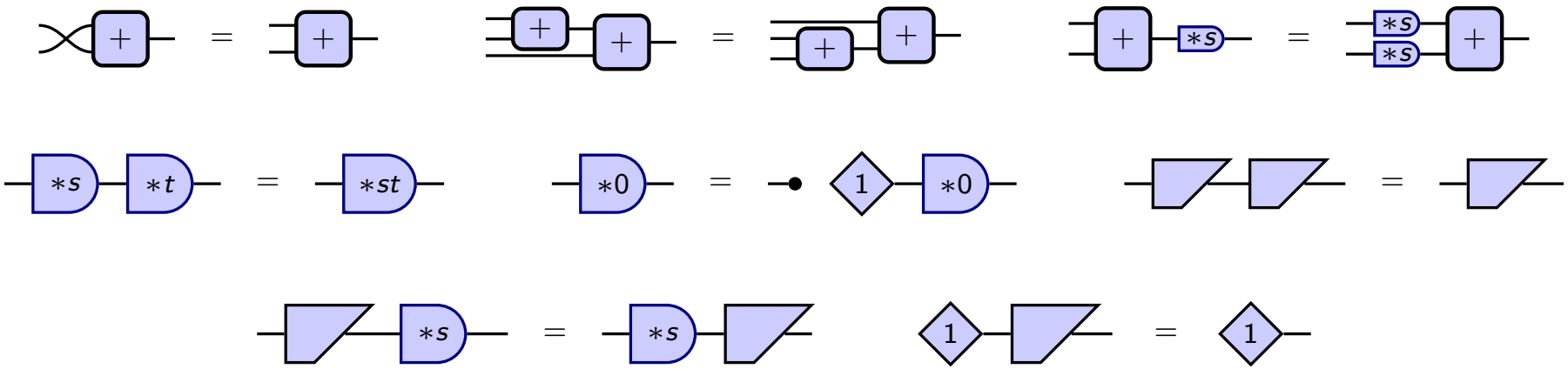

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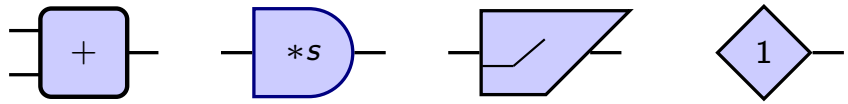

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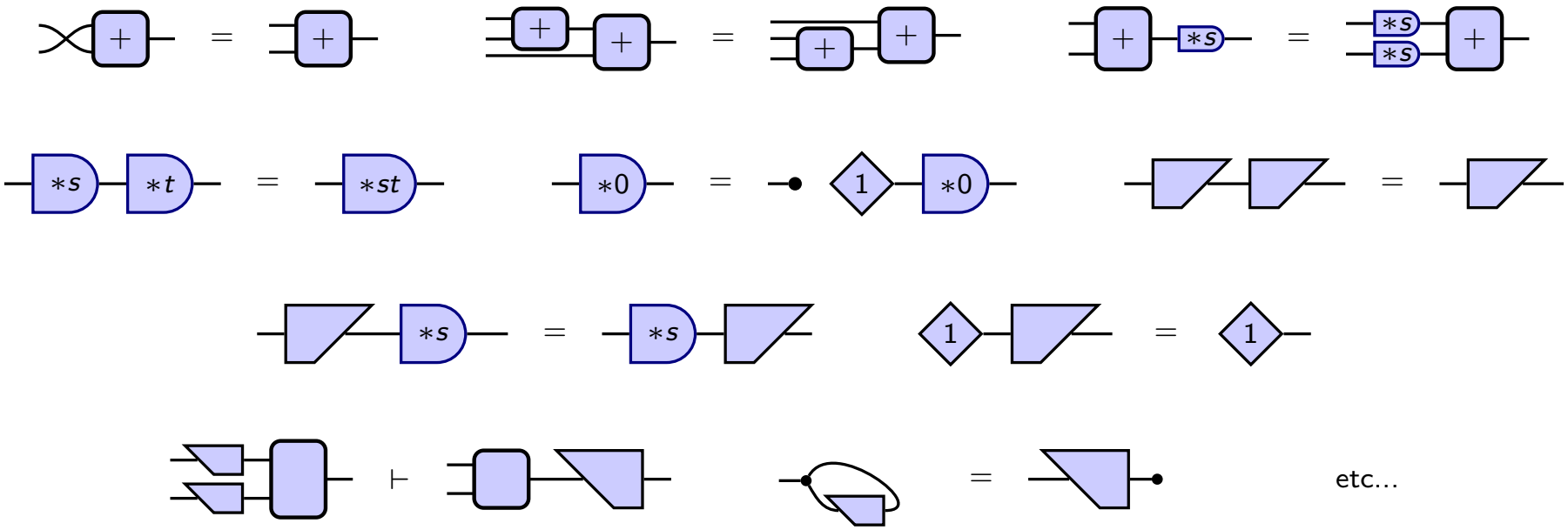

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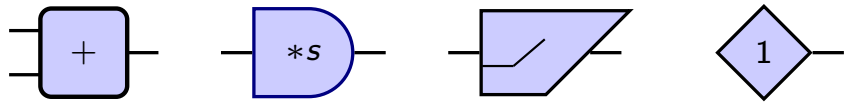

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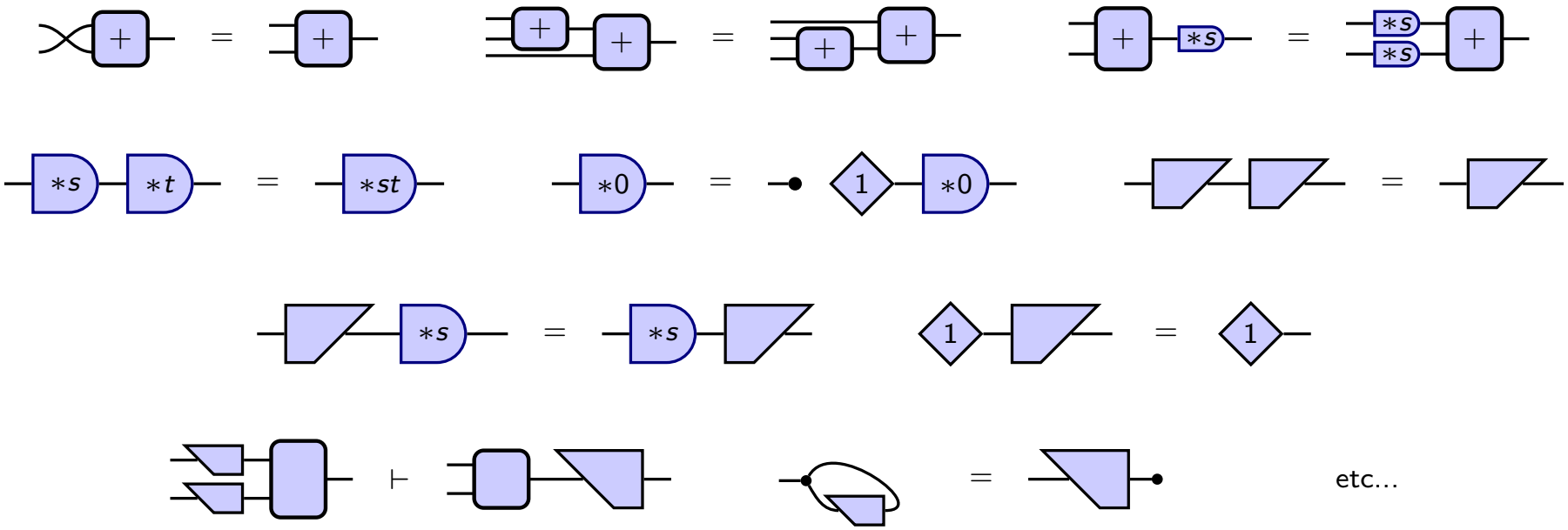
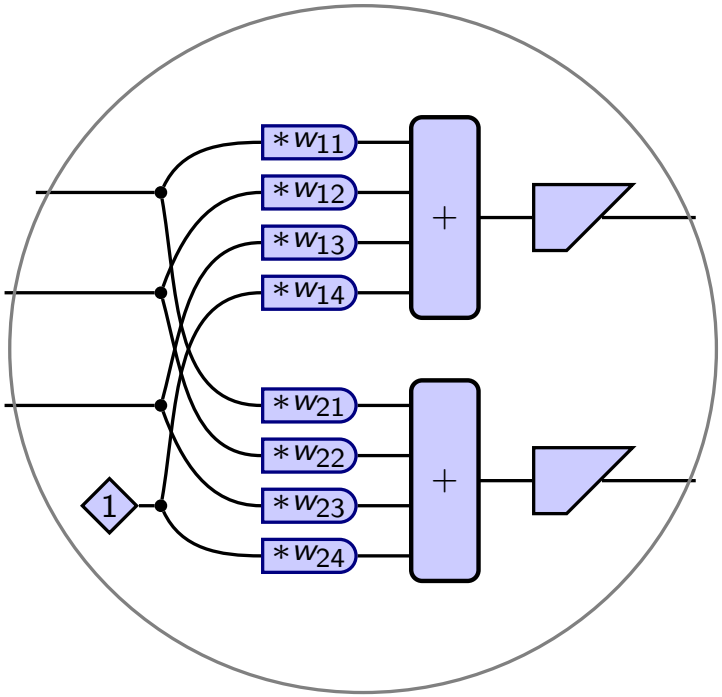


One could presumably work out a sound and complete set of axioms.

Game: neural relations

Users play with neural networks:



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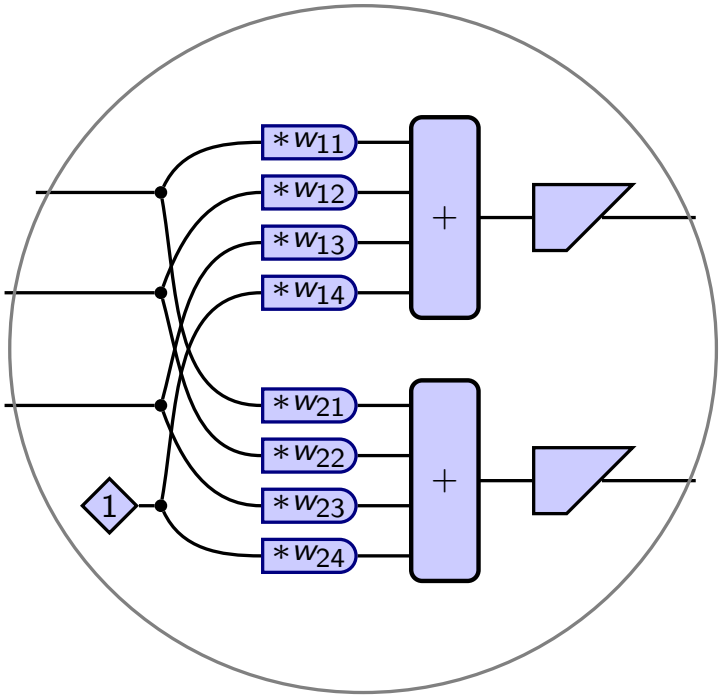
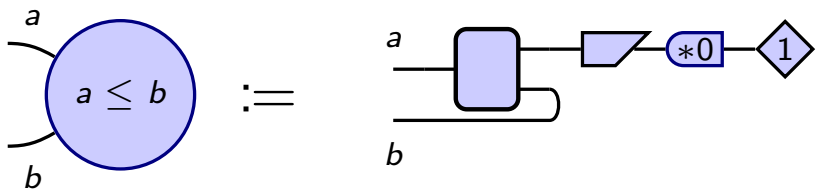


And invent other relations if they feel like it:



Outline

Can we make this real?

I'd love to see something like this be implemented.

- Users can get comfortable with one platform for many "games".
- The background math is complete.
  - Pick types T, pick ajax functor $P\colon \mathbb{C}\mathrm{ospan}_{\mathsf{T}}^{\mathrm{co}} \longrightarrow \mathbb{P}\mathrm{oset}$.
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  - Playing the game involves wiring up cells and seeing how they relate.
  - There are as many games as their are regular categories.
  - There are of course morphisms of games $(F\colon \mathsf{T} \to \mathsf{T}', F^\sharp\colon P \to P' \circ F)$.

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  - Pick types T, pick ajax functor $P\colon \mathbb{C}\mathrm{ospan}_{\mathsf{T}}^{\mathrm{co}} \longrightarrow \mathbb{P}\mathrm{oset}$.
  - Each pair (T, P) constitutes a different game.
  - Playing the game involves wiring up cells and seeing how they relate.
  - There are as many games as their are regular categories.
  - There are of course morphisms of games $(F\colon \mathsf{T} \to \mathsf{T}', F^\sharp\colon P \to P' \circ F)$.
- Seems like implementation would benefit a lot from dependent types.

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Thanks! Questions and comments welcome!!