

# Polynomial Functors and Shannon Entropy

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David I. Spivak



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# Outline

## 1 Introduction

- Why am I here?
- Working in the **Poly** ecosystem
- Plan of the talk

## 2 Background on Poly

## 3 Distributive functors and entropy

## 4 Conclusion

# Why am I here?

We're here to learn from each other. But what is learning?

- Somehow out of all the information out there, some of it *sticks*.
- We develop frameworks by which to *store* information.
- I'm interested in how intelligence and learning function.
- So I study how knowledge is stored and transferred in databases and...
- ...how dynamical systems interact to adapt and learn (e.g. in DNNs).

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Entropy has been put forward as an approach to intelligence and learning.

- Life can be understood as a *dissipative system*, spraying entropy.
- It does so while packing negentropy—organization—into itself.
- Polani's *empowerment* and Freer's *causal entropic forces*...
- ...are entropy-based approaches to intelligent behavior.

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I only seem to understand things when they're written categorically.

- I've been trying to understand “what entropy really is.”
- The Baez-Fritz-Leinster conception of entropy is great,...
- ...but I want to connect it in with dynamical systems or databases.

# The overwhelming abundance of Poly

In January 2020 I fell in love with a category called **Poly**.

- Its applications subsume everything I'd done with categ'l databases...
- ...and everything I'd done with interacting dynamical systems.
- It's used in functional programming, type theory, higher cat'y theory.

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- Its applications subsume everything I'd done with categ'l databases...
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- It's used in functional programming, type theory, higher cat'y theory.

But it's not just very applicable, it's also very highly-structured.

- Coproducts and products that agree with usual polynomial arithmetic;
- All limits and colimits;
- At least three orthogonal factorization systems;
- A symmetric monoidal structure  $\otimes$  distributing over  $+$ ;
- A cartesian closure  $q^p$  and monoidal closure  $[p, q]$  for  $\otimes$ ;
- Another nonsymmetric monoidal structure  $\triangleleft$  that's duoidal with  $\otimes$ ;
- A left (Meyers?)  $\triangleleft$ -coclosure  $\left[ \begin{smallmatrix} - \\ - \end{smallmatrix} \right]$ , meaning  $\mathbf{Poly}(p, q \triangleleft r) \cong \mathbf{Poly}\left(\left[ \begin{smallmatrix} f \\ p \end{smallmatrix} \right], q\right)$ ;
- An indexed right  $\triangleleft$ -coclosure, i.e.  $\mathbf{Poly}(p, q \triangleleft r) \cong \sum_{f: p(1) \rightarrow q(1)} \mathbf{Poly}(p \overset{f}{\triangleleft} q, r)$ ;
- An indexed right  $\otimes$ -coclosure (Niu?), i.e.  $\mathbf{Poly}(p, q \otimes r) \cong \sum_{f: p(1) \rightarrow q(1)} \mathbf{Poly}(p \overset{f}{\otimes} q, r)$ ;
- At least ten monoidal structures in total;
- $\triangleleft$ -monoids generalize  $\Sigma$ -free operads;
- $\triangleleft$ -comonoids are exactly categories; bicomodules are data migrations.

See “A reference for categorical structures on **Poly**”, arXiv: 2202.00534 2 / 15

# Entropy in terms of Poly

Today I'll tell you how entropy looks from the **Poly** point of view.

- I'll show how to think of objects in **Poly** as empirical distributions.
- I'll show that there are distributive monoidal functors

$$\mathbf{Poly}^{\text{Cart}} \xrightarrow{p \mapsto \dot{p}y} \mathbf{Poly} \xrightarrow{p \mapsto (p(1), \Gamma(p))} \mathbf{Set} \times \mathbf{Set}^{\text{op}}$$

sending  $p \in \mathbf{Poly}^{\text{Cart}}$  to an invariant  $h(p) := (A, B) \in \mathbf{Set} \times \mathbf{Set}^{\text{op}}$ .

- The Shannon entropy can then be extracted:  $H(p) = \log(A/\sqrt[A]{B})$ .
- Properties of entropy follow from the distributive monoidality of  $h$ .

# Plan

The plan for the rest of the time is as follows:

- Give background on polynomial functors.
- Explain  $h: \mathbf{Poly}^{\mathbf{Cart}} \rightarrow \mathbf{Set} \times \mathbf{Set}^{\mathbf{op}}$  and its relation to entropy.
- Talk about generalizations and future work.
- Conclude.

# Outline

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## 2 Background on Poly

- The category **Poly**
- Distributive monoidal structure
- Other theoretical aspects

## 3 Distributive functors and entropy

## 4 Conclusion

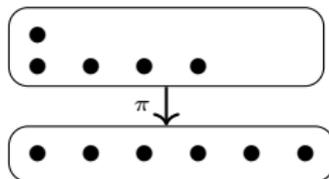
# What is a polynomial?

Here's how we'll think about polynomial functors.

**Algebraic**

$$y^2 + 3y + 2$$

**Bundle**



**Corolla forest**



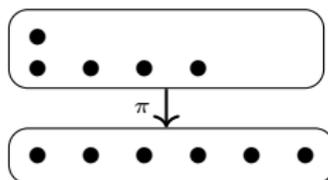
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You can think of the bundle as an empirical distribution:

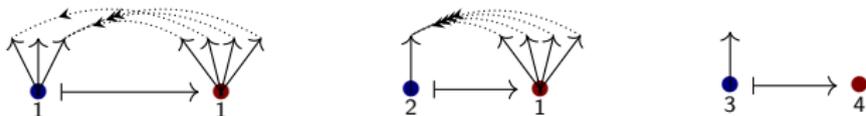
- The first outcome was drawn twice; the next three once; the rest never.
- It corresponds to the distribution  $(\frac{2}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, 0, 0)$ .

# What is a morphism of polynomials?

Let  $p := y^3 + 2y$  and  $q := y^4 + y^3 + y + 1$



A morphism  $p \xrightarrow{\varphi} q$  sends  $p$ -outcomes to  $q$ -outcomes, interpreting draws:



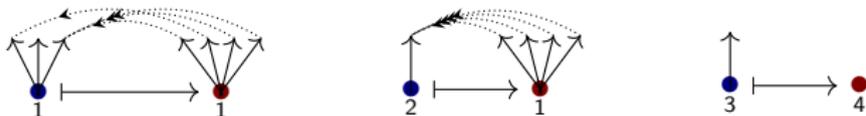
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It is *Cartesian* iff each map on draws is a bijection. ( $\mathbf{Poly}^{\text{Cart}}(p, q) \cong 6$ .)

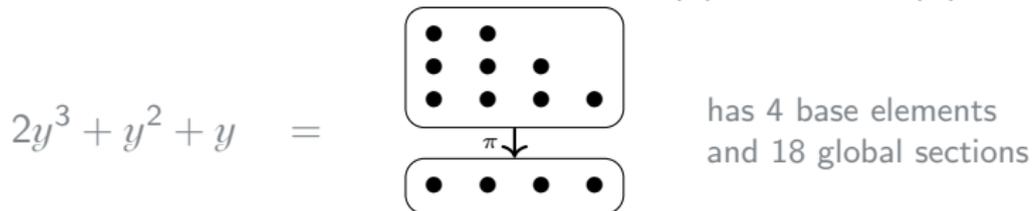
## Fundamental invariants

We will be interested in two fundamental invariants of a polynomial.

- From the bundle POV, these would be *base* and *global sections*.
- So if  $p$  is represented by  $E \rightarrow B$ , these are  $B$  and  $\mathbf{Set}_{/B}(B, E)$ .
- In terms of polynomials these are

$$p(1) \cong \mathbf{Poly}(y, p) \quad \text{and} \quad \Gamma(p) := \mathbf{Poly}(p, y).$$

- E.g. for the following bundle these are  $p(1) \cong 4$  and  $\Gamma(p) \cong 18$ .



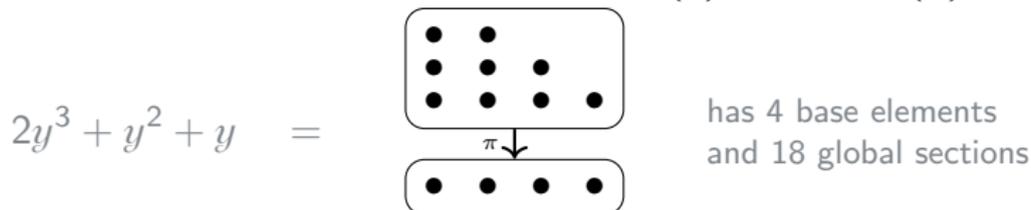
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These are functorial in opposite directions:

$$\mathbf{Poly} \xrightarrow{p \mapsto (p(1), \Gamma(p))} \mathbf{Set} \times \mathbf{Set}^{\text{op}}$$

This functor has a right adjoint, but we won't use that.

# The distributive monoidal structure $(\mathbf{Poly}, 0, +, y, \otimes)$

The category **Poly** is distributive monoidal.

- The usual sum of two polynomials is their coproduct;  $0$  is initial.
- There is another operation  $\otimes$  called *Dirichlet product*. Formula:

$$p \otimes q := \sum_{(i,j) \in p(1) \times q(1)} y^{p[i] \times q[j]}$$

These are very simple bundle-wise: sum & product of base and total space:

$$\begin{array}{ccc}
 E_1 & E_2 & E_1 + E_2 \\
 \downarrow & \downarrow & \downarrow \\
 B_1 & B_2 & B_1 + B_2
 \end{array}
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and

- These clearly distribute:  $p \otimes (q_1 + q_2) \cong (p \otimes q_1) + (p \otimes q_2)$ .
- Soon we'll see how the fundamental invariants respect these oper'ns.

## Derivatives and the bifibration

The derivative of a polynomial functor is another polynomial functor.

- Write  $\dot{p}$  for the derivative with respect to  $y$ ; consider also  $\dot{p}y$ .

$$\dot{p} = \sum_{i \in p(1)} \sum_{d \in p[i]} y^{p[i] \setminus \{d\}} \quad \text{and} \quad \dot{p}y \cong \sum_{i \in p(1)} p[i]y^{p[i]}$$

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Neither of these is functorial in **Poly**, but *they are functorial* in **Poly**<sup>Cart</sup>.

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The last theory we'll need is the bifibration **Poly**  $\rightarrow$  **Set**.

- The functor  $p \mapsto p(1)$  is a monoidal \*-bifibration. All we need:
- For any  $f: A \rightarrow B$ , we have  $\mathbf{Poly}_A(f^*q, p) \cong \mathbf{Poly}_B(q, f_*p)$ .

$$f^*q := \sum_{a \in A} y^{q[fa]} \quad \text{and} \quad f_*p := \sum_{b \in B} y^{\sum_{a \rightarrow b} p[a]}$$

As we lump outcomes together, we add up the draws.

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## 3 Distributive functors and entropy

- Distributive functor  $\mathbf{Poly}^{\mathbf{Cart}} \rightarrow \mathbf{Set} \times \mathbf{Set}^{\mathbf{op}}$
- Entropy and entropy density

## 4 Conclusion

## $p \mapsto \dot{p}y$ is distributive

We're now ready to get to work on how all this relates to entropy.

- The approach is to extract two invariant sets from any polynomial.
- This process is “good” in that it is a distributive monoidal functor.
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So first, we want to see that  $p \mapsto \dot{p}y$  is distributive monoidal.

- The derivative is linear,  $(p \dot{+} q) = \dot{p} + \dot{q}$ , and so is  $p \mapsto \dot{p}y$ .
- So  $p \mapsto \dot{p}y$  preserves coproducts  $+$ . It also preserves  $\otimes$ :

$$\begin{aligned}
 (\dot{p}y) \otimes (\dot{q}y) &\cong \sum_{i \in p(1)} p[i]y^{p[i]} \otimes \sum_{j \in q(1)} q[j]y^{q[j]} \\
 &\cong \sum_{(i,j) \in p(1) \times q(1)} p[i] \times q[j]y^{p[i] \times q[j]} \\
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So  $(\mathbf{Poly}^{\mathbf{Cart}}, 0, +, y, \otimes) \xrightarrow{p \mapsto \dot{p}y} (\mathbf{Poly}, 0, +, y, \otimes)$  is distributive monoidal.

## Fundamental invariants are distributive

The fundamental invariants  $p \mapsto (p(1), \Gamma(p))$  are also distributive

$$(\mathbf{Poly}, 0, +, y, \otimes) \xrightarrow{p \mapsto (p(1), \Gamma(p))} (\mathbf{Set} \times \mathbf{Set}^{\text{op}}, (0, 1), +, (1, 1), \otimes)$$

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- $\mathbf{Set} \times \mathbf{Set}^{\text{op}}$  has coproducts:  $(A_1, B_1) + (A_2, B_2) \cong (A_1 + A_2, B_1 \times B_2)$ .
- It has another symmetric monoidal structure with unit  $(1, 1)$ :

$$(A_1, B_1) \otimes (A_2, B_2) := (A_1 \times A_2, B_1^{A_2} \times B_2^{A_1})$$

- And these distribute “because”  $B^{A_1 + A_2} (B_1 B_2)^A \cong (B^{A_1} B_1^A) (B^{A_2} B_2^A)$ .

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- And these distribute “because”  $B^{A_1 + A_2} (B_1 B_2)^A \cong (B^{A_1} B_1^A) (B^{A_2} B_2^A)$ .

Why do the fundamental invariants (as a pair) preserve  $+$  and  $\otimes$ ?

- We have  $(p + q)(1) \cong p(1) + q(1)$  and  $\Gamma(p + q) \cong \Gamma(p) \times \Gamma(q)$ .
- This says they preserve  $+$ . One also checks they preserve  $\otimes$ :

$$(p \otimes q)(1) \cong p(1) \times q(1) \quad \text{and} \quad \Gamma(p \otimes q) \cong \Gamma(p)^{q(1)} \times \Gamma(q)^{p(1)}$$

## Taking stock

Let's denote the composite of our distributive functors by  $\hat{h}$ :

$$\begin{array}{ccc}
 \mathbf{Poly}^{\mathbf{Cart}} & \xrightarrow{p \mapsto \dot{p}y} & \mathbf{Poly} & \xrightarrow{p \mapsto (p(1), \Gamma(p))} & \mathbf{Set} \times \mathbf{Set}^{\mathbf{OP}} \\
 & & \parallel & & \nearrow \\
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- The claim is that  $\hat{h}$  extracts everything you need to calculate entropy.
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Define a real number  $L(A, B) := \frac{\log(\#A^{\#A}) - \log(\#B)}{\#A}$ . Then:

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### Theorem

*Let  $p$  be a polynomial, considered as a probability distribution  $P$ , and let  $H(P)$  be its Shannon entropy. Then we have*

$$H(P) = L(\hat{h}(p))$$

# The categorical partition function and entropy

I'm unfamiliar with the thermo picture. Joint with James Dama:

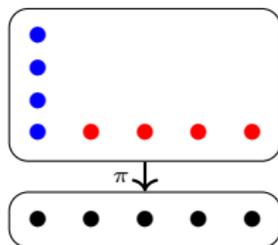
- One should think of Shannon entropy as an *entropy density*.
- The thermo picture defines a *partition function*  $\Omega$  for distributions.
- For  $p \in \mathbf{Poly}$  with  $h(p) = (A, B)$ , this would be  $\Omega_p := \frac{A^A}{B}$ .
- Then the extensive entropy of  $p$  is given by  $E(p) := \log \Omega(p)$ .
- And the Shannon entropy of  $p$  is the density  $H(p) := E(p)/A$ .

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Let's see a worked example. Consider the bundle for  $p := y^4 + 4y^1$ :



- We find  $\dot{p}y = 4y^4 + 4y$ , so  $h(p) = (4 + 4, 4^4) = (8, 4^4)$ .
- So  $\Omega_p = \frac{8^8}{4^4}$ , Ext've:  $E(p) = \log \Omega_p = 16$ , Shannon:  $H(p) = 16/8 = 2$ .

## Consequences of distributivity

The fact that  $h$  preserves  $\otimes$  and properties of log immediately give us

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- Again joint with James Dama.
- Suppose you write  $p$  as a sum,  $p := \sum_{a \in A} p_a$ .
- This is the same as giving a function  $f: p(1) \rightarrow A$ .
- Recall that from this we get  $f_*p \in \mathbf{Poly}_A$ , lumping distributions.

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It follows from the fact that  $h$  preserves sums that:

$$\Omega_p = \Omega_{f_*p} \times \prod_{a \in A} \Omega_{p_a} \quad \text{and} \quad E(p) = E(f_*p) + \sum_{a \in A} E(p_a)$$

The usual “chain rule” for Shannon entropy follows directly from this.

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There is (at least one) interpretation of entropy within **Poly**.

- Objects in **Poly** can be viewed as empirical distributions.
- There is a  $(+, \otimes)$ -preserving functor  $h: \mathbf{Poly}^{\text{Cart}} \rightarrow \mathbf{Set} \times \mathbf{Set}^{\text{op}}$ .
- Theorem: if  $(A, B) := h(p)$  then  $H(p) = \log(A/\sqrt[A]{B})$ .
- So all the entropy-relevant data of  $p$  is encapsulated in two sets.

# Summary

Entropy and **Poly** are amazing and have overlapping applications.

- Entropy is well-known throughout the scientific and technical world.
- **Poly** is the most highly-structured category I've ever seen.
- Both have applications to interacting dynamical systems.

There is (at least one) interpretation of entropy within **Poly**.

- Objects in **Poly** can be viewed as empirical distributions.
- There is a  $(+, \otimes)$ -preserving functor  $h: \mathbf{Poly}^{\text{Cart}} \rightarrow \mathbf{Set} \times \mathbf{Set}^{\text{op}}$ .
- Theorem: if  $(A, B) := h(p)$  then  $H(p) = \log(A/\sqrt[A]{B})$ .
- So all the entropy-relevant data of  $p$  is encapsulated in two sets.

Entropy still feels somehow foreign to me.

- Hopefully, having different categorifications will help clarify it.
- I still have hope that it will someday bond with the dynamics of **Poly**.

*Thanks! Comments and questions welcome...*