# Category theory is living language

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# Outline

1 **Introduction**
- Why am I here?
- Sense-making
- Accounting systems
- Plan for the talk

2 What is working language?

3 ACT as living language

4 Conclusion

# Why am I here?

In 2007, I read *The Moment of Complexity* by Mark C. Taylor

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I wanted to do something, to help the world coordinate.

- Each person and organization structures its experience of the world.
- My neuron pattern and yours are more different than our fingerprints.
- And these are quite different than the database schemas found in orgs.
- Despite these massive differences, we are able to communicate! How?
- For the world to solve big problems, we must communicate better.
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I'm here because I want to say how I understand this process now.

- How can we think about the power of language for living systems?
- How is category theory a language, and in what sense is ACT living?

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Consider a snapshot of two math students, both wanting to succeed:

- Student A is faithfully copies down what the teacher says.
- Student B seems to be doing the opposite: ...
- ...clearly frustrated, arguing with the teacher, "but then why XYZ??"
- Suddenly student B says "Oh!! Is it because ABC??"
- B relaxes, having made sense. Later: B does better than A on tests.

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*Making sense* of things takes work, but it produces sense!

- The work of trying to make things fit together results in new sense.
- We can solve harder problems if we make better sense of things.

# Accounting systems

How do collectives make sense?

- Human society, CT-ists, a family, a multi-cellular organism, ...
- ...all of these are collectives that benefit from making collective sense.
- To work together: minimize friction and stepping on each others' toes.

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- Arithmetic accounts for the flow of quantities, as in finance.
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One reason the world is losing sense is we distrust each others' accounts.

- Languages in which to give accounts are becoming more valuable.

# Plan for the talk

During the remainder of the talk, I'll

- Consider languages and how they function,
- Talk about ACT as living language, and
- Conclude with a summary.

# Outline

1 Introduction

2 What is working language?
- The structure and dynamics of working language
- Matter and pattern
- Compositionality

3 ACT as living language

4 Conclusion

# The structure and dynamics of working language

Language *works*: it leads to the displacement of material objects in space.

- If I say "pass the salt", $10^{20}$ atoms, i.e. of arm, salt shaker...
- ...move through space, and some salt crystals arrive in my soup.
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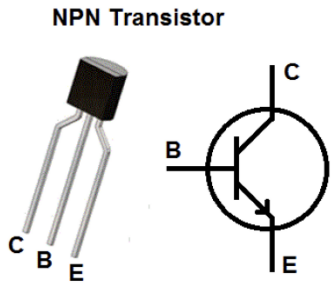
But how does it actually *work*?

- Not naturally captured by *physics*, e.g. strong, weak, EM, gravity.
- Maybe we need different abstractions to see why it is so powerful.
- Let's go deeper.

# Matter and pattern

*Matter* and *Pattern* are fundamental notions.

- Etymology: mother and father.
- Material world vs. concepts, mathematics.
- The transistor as *symbol*: matter&pattern



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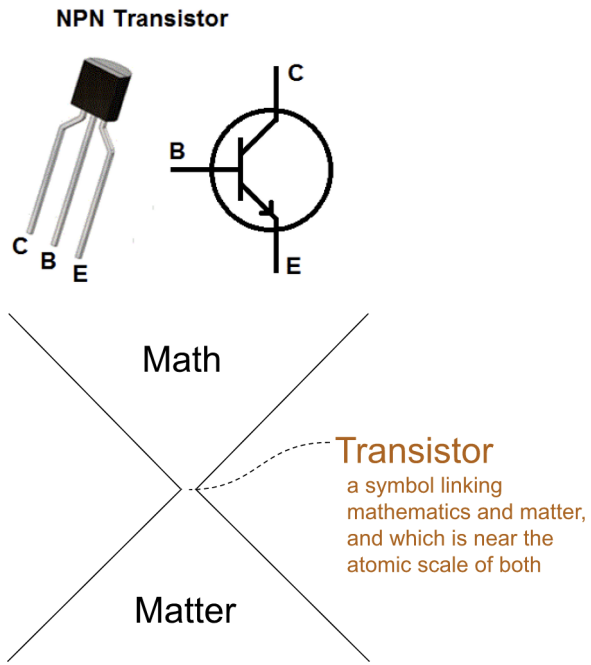
- It's close to the atomic scale in both:
  - Two transistors make NAND $\rightsquigarrow$ all logic
  - A transistor is $\sim$ 10 Si atoms in length
- All math can be done in Agda, ...
- ...implemented in physics via the transistor.



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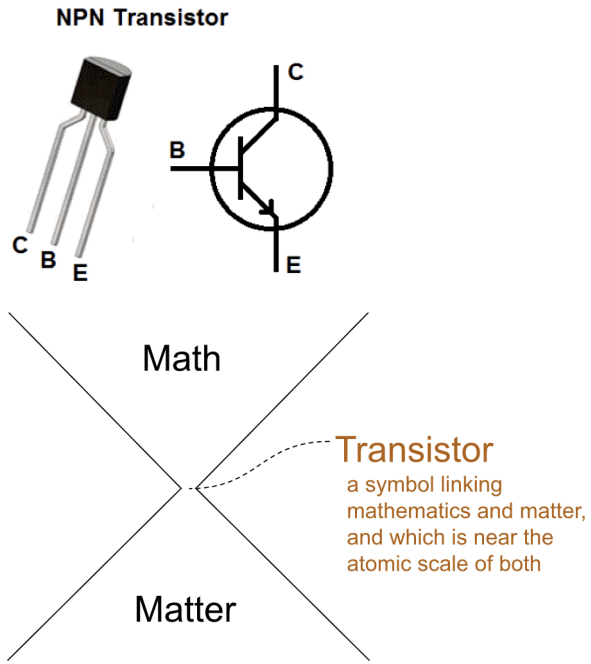


Other symbols—condensed matter&pattern composites—include:

- DNA: acidic 3D molecule & ordered seq. of four letters coding for AAs
- Signature: pen moving across paper & token of agreement.
- Economic transaction: credit-card swipe & token of exchange.
- Grandma neuron: cell in brain & represents your grandma.

Symbols are at the base of language.

# Compositionality

Compositionality of the symbols is what makes language work.

- The transistor as matter&pattern, in isolation, wouldn't be enough.
- It's the fact that congruence remains even as we compose.
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Compositionality allows the language to actually work.

- Compose transistors to get logic gates, adder circuits, latches, CPUs.
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Each of these *languages* has its own grammar, ways to compose.

- So language works by having the patterns track the matter...
- ...so symbols can be meaningfully composed via the grammar.

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1 Introduction

2 What is working language?

3 ACT as living language
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- Using ACT to make sense
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# ACT as living language

Language comes alive when it is used to direct material change in the world.

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- CT accounts for structural similarities that cross disciplines.
- We articulate common structures and mappings that preserve them.
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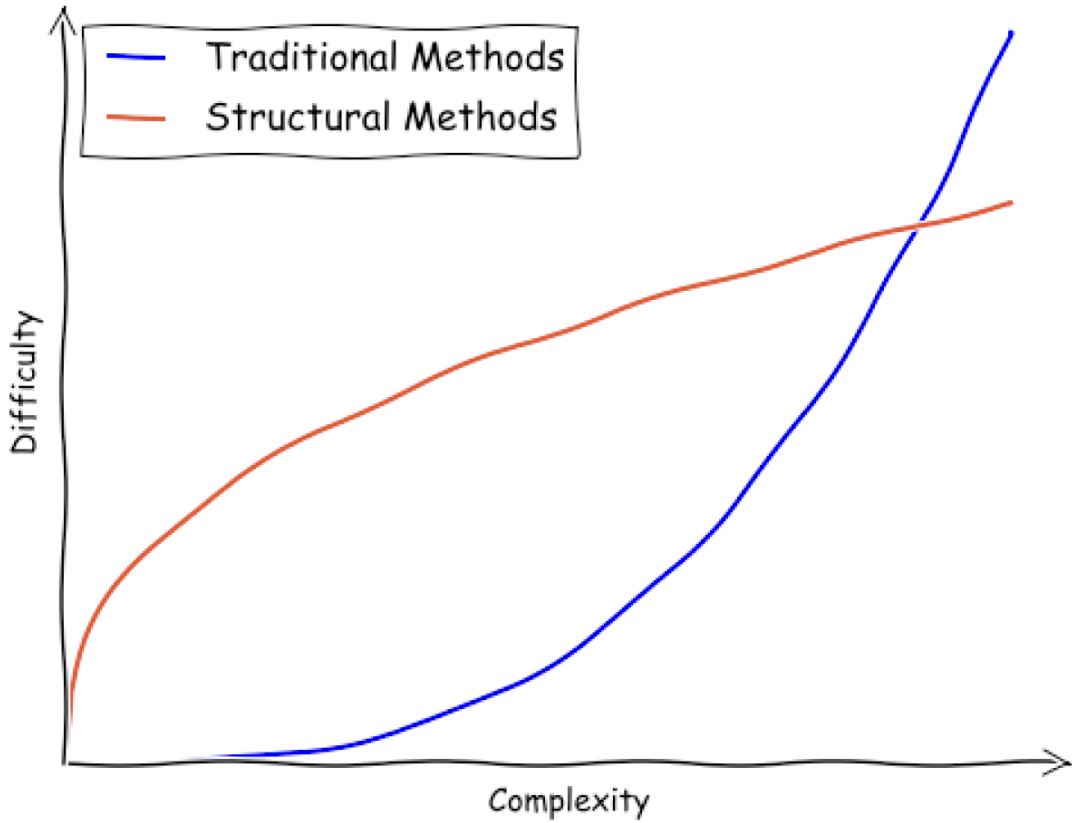
ACT is living language when the mathematics is used in the real world.

- People implement ACT as quantum compilers, database systems, programming languages, dimension reduction algorithms, deep learning systems, multi-physics simulators, epidemiology modeling tools, etc.
- Cat'l implementations are usually more maintainable and interoperable.
- They're also usually less buggy, since the CT manages the corner cases.

# Spencer's graph

I love this graph, due to Spencer Breiner (NIST):

## Categorical Engineering




As things become more complex, structural methods become necessary.

# Using ACT to make sense

The world is becoming more complex, so it's harder to make sense of.

- There are more interconnections, and information moves faster.
- This causes more change in less time. How do we track it?
- Making sense means tracking the right variables.

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Category theory helps us see the big picture and the little details.

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- It lets us link them, even across language barriers.
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By applying CT to the world, we are making sense of it.

- Language connects material conditions to conceptual understanding.
- ACT does this too, at least in certain domains (quantum, CS, scientific modeling...)
- Every new application of CT brings a little more sense to the world: ...
- ...You track the right variables to enable good interoperability, etc.

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- Summary
- Topos is hiring!

# Summary

Making sense of the world is important for everyone.

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ACT folks are involved in a collective sense-making enterprise.

- Encoding features of the world into CT has a tangible effect.
- I propose we aim for elegance—good sense—and accountability.

*Thanks! Comments and questions welcome...*

# Topos is hiring!

Topos Institute is hiring!

- Topos is a nonprofit research institute located in Berkeley.
- Our mission is to use CT to shape technology for public benefit.
- Our work includes research, tool-building, and education.

We're looking for:[1]

- Research software engineers,
- Operations and financial assistants,
- Educators that can explain our work, and
- Summer Research Assistants.
- We're very interested in inclusivity and representation.

Please get in touch at `info@topos.institute` if you're interested!

[1] `https://topos.institute/contact`