How can math promote life?

David I. Spivak

TOPOS INSTITUTE

Calgary Mathematics & Philosophy Lecture
2024 March 07

Outline

NeverEnding Story

The book *The NeverEnding Story* has a special property:

- It is like a fractal, designed so the reader to become part of the story.
- If we care, we become a character in it and can influence how it goes.

This talk is an attempt to do the same: to tell a story about us, right here.

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We're here in this room. How did it come to be? What is the purpose?

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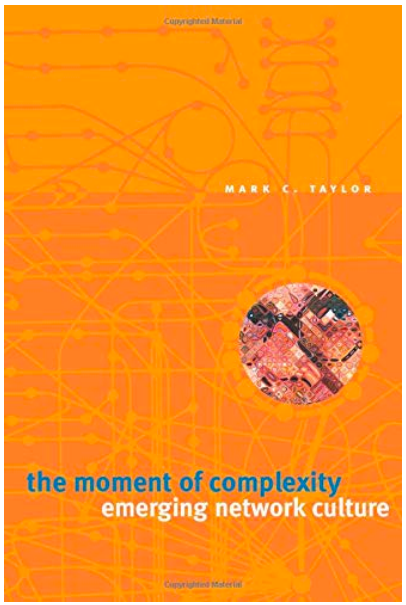

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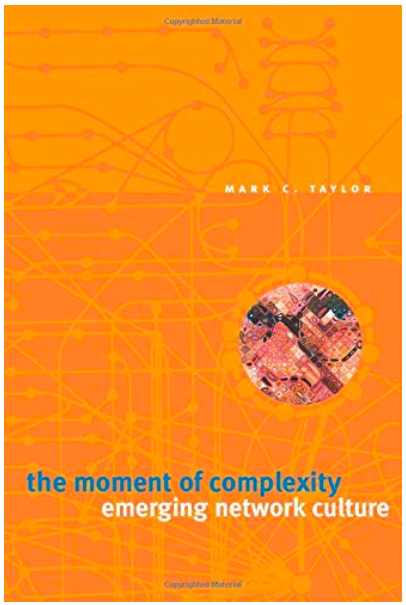

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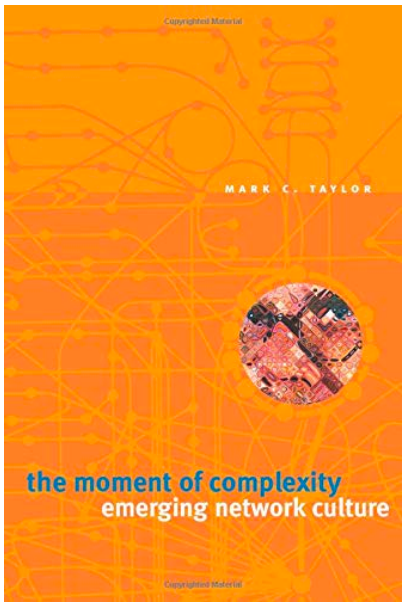

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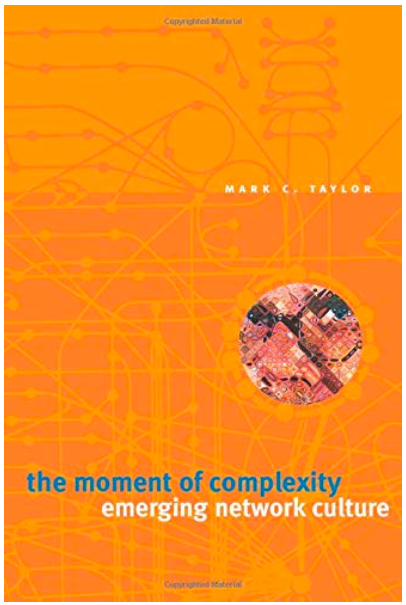

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Together, let's consider the question: *How can math promote life?*

- We're asking in the context of a math and philosophy lecture.
- Caveat: I'm not a trained philosopher.
- Check philosophical claims in your own experience or with an expert.

Bridging ancient conceptions of the good

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- Plato: good = mathematical order and harmony.
- Aristotle: good = human flourishing.
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What I mean by *sense* is not raw perception: it's orderly, not chaotic.

- Think sense of humor, direction, danger, opportunity.
- It is a cultivated "ecology of mind" for tracking the right variables.
- When we have a good sense of something, we navigate it with ease.

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Ecology of mind is sense, and it is a Plato-Aristotle good.

Accountability

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- "The ability to account": We can give and receive accounts.
- We can articulate our experience, and experience what's articulated.
- Corollary: "Being accountable", being receptive to feedback.

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Accountability is core to bridging mathematics and human flourishing.

- I'll explain how mathematical fields are well-ruled accounting systems.
- J. Butler says the moral dialectic is "giving an account of oneself."
- In law, all the above aspects of accountability are quite clear.
  - Strict rules govern how accounts are to be given ("objection!")
  - Feedback from the system is enforced by the collective (wardens).
- In math and polis, we must explain our steps and correct mistakes.

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I want this talk to be exemplary of its own assertions.

- I want it to be accountable and lead to collective sense-making.
  - I'll provide math to explain what I'm saying, so you can check it.
  - The ideas will face your feedback during the Q&A.
- Math promotes life when it helps us make sense of it together.

Dynamic arrangements

We're constantly adjusting to fit with our surround.

- The reason for accountability is so that we can "distribute the load".
- Think of some suspended fabric; someone drops a lead ball into it.
- Each knot of fabric sends force to its neighbor saying "come this way!"
- We collectively solve problems through effective accounting.

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- Later, the arrangement will change and I'll dialogue with you.
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- We can promote life by making sense of it.
- Life involves dynamic arrangements of interacting sense-makers.
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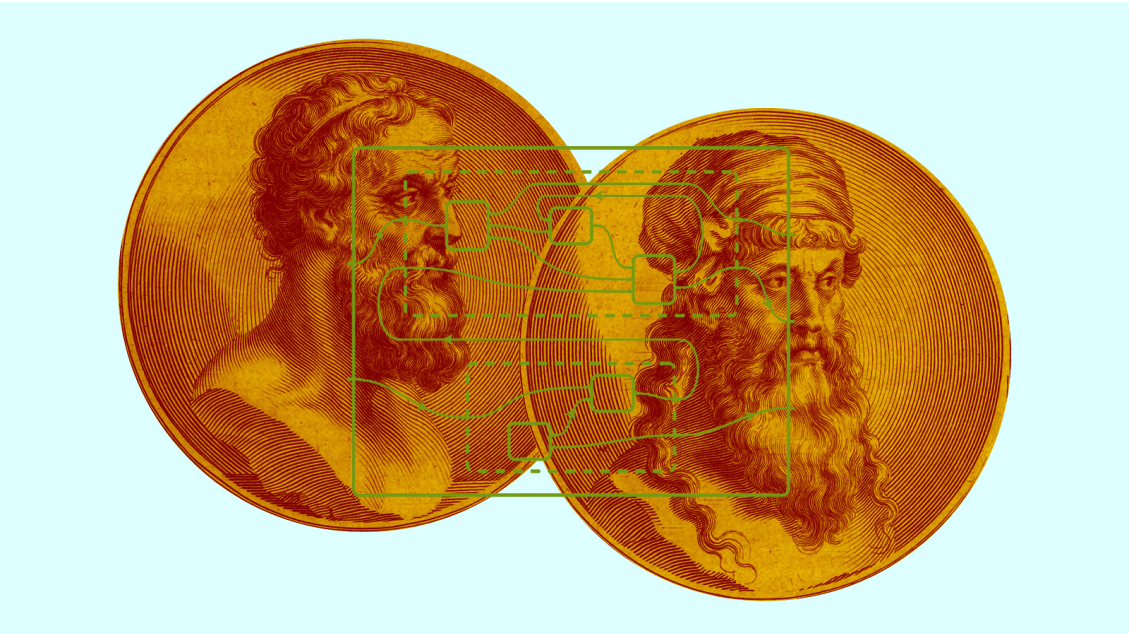
The math we'll use is called *category theory*. Let's get started!

Plan for talk



The plan for the rest of the talk is as follows.

- I'll explain the idea of math fields as well-ruled accounting systems.
- I'll discuss *category theory*, which accounts for structure and analogy.
- I'll showcase some math about interfaces and dynamic arrangements.
- I'll conclude with a summary.

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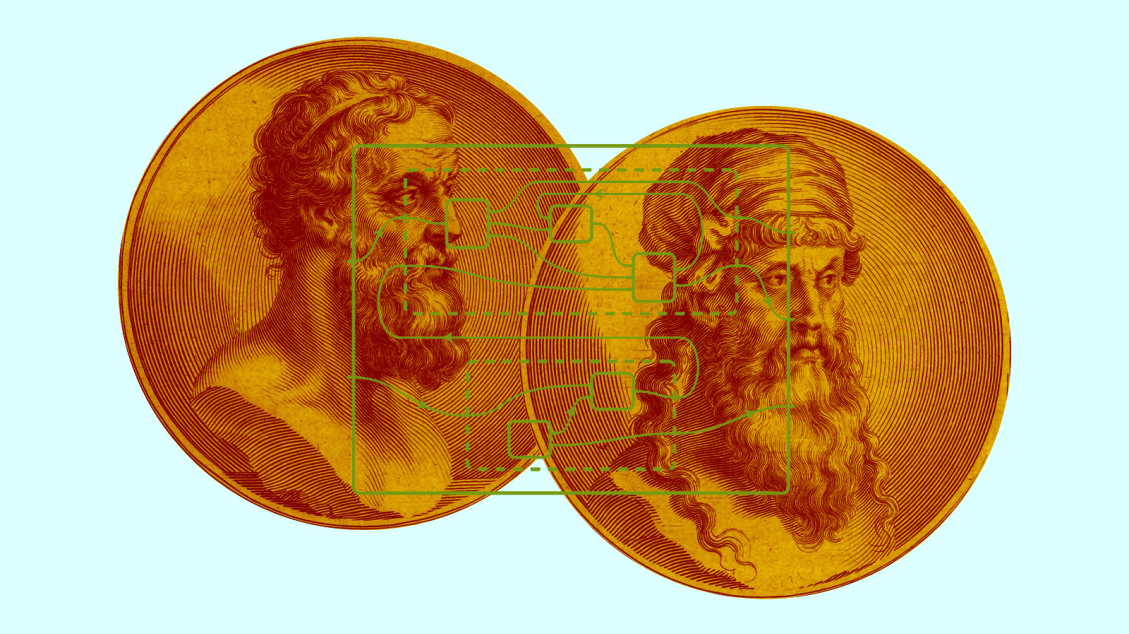

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I know the slides are full of text; I hope they're clear and accountable.

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Another look at sense-making

All sense-making is collective sense-making.

- A human's cells work in concert together to form their sense.
- A group of us work in concert as a corporation.
- In both cases, alignment offers the possibility of *concerted effort*.

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But how is alignment achieved? How is (collective) sense made?

- Consider a snapshot of two math students, both wanting to succeed:
- Student A is faithfully copies down what the teacher says.
- Student B seems to be doing the opposite: ...
- ...clearly frustrated, arguing with the teacher, "but then why XYZ??"
- Suddenly student B says "Oh!! Is it because UVW??"
- Something has "clicked" and B relaxes, having *made sense*.
- Later: B does better than A on tests, catches the teacher's typos, etc.

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What is the phenomenology here?

- When things don't make sense, the accounting doesn't line up.
- The frustration is a dissonance between participants in the collective.
- When accounts settle and alignment is found, click! like a ratchet.

Phenomenology, rules, and the unaccounted-for

Internet Encyclopedia of Philosophy, "Phenomenological Psychology"

Phenomenology is the theory (ology) of phenomena.

- It is how we'll talk about a given sense we have.
- We can build theories to account for our sense.
- A phenomenology is an accounting system.
- Soon we will talk about math as well-ruled accounting.

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I want to talk about rules in three ways.

- Rules are lines; they delineate. Rule as straight-edge.
- Rules are measures; they value. Rule as size-standard.
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We must remember that our accounting system is always partial.

- It abstracts over certain differences; these are left unaccounted-for.
- "Whereof one cannot speak thereof one [will] remain silent."
- Our mathematics will always miss things: it evolves as they're found.
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Is this really possible? Could new math be invented today? Let's consider.

Mathematical fields as well-ruled accounting systems

Mathematical fields can be understood as *well-ruled accounting systems*.

- Arithmetic accounts for the flow of quantities, as in finance.
- Hilbert spaces account for the states of elementary particles, as in QM.
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How are they well-ruled systems for accounting?

- Pick one of the above, and check along with me.
- Is the math a kind of language by which one can give accounts?
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Mathematics is not static; it is constantly being invented.

- Every field of mathematics was invented and developed by people.
- Newton wanted to account for relative rates of change.
- Frege wanted to account for logical propositions.
- We invent math to account for regularity we notice in phenomena.

Reassessing the goodness of mathematical rules

Mathematics is very powerful, harnessing and directing energy.

- Math lets people coordinate their efforts across time and space.
- Break up problems, solve pieces independently, and integrate results.
- Etymologically, correct means right with, straight with, aligned with.
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Well-ruled accounting harnesses and directs energy in various ways:

- as a test harness, it helps with auditing and regulation;
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How can we address the tendency toward imperialism?

- The word "ruled" is not entirely positive. Are we ruled by GDP?
- The word "harness" is a bit scary. What is our energy going toward?
- The empirical power from applying math reminds of imperial power.
- Whereof we don't account, we leave in silence.
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But the math we have isn't going anywhere, so what do we do?

Accounting for perspectives and analogy

All math was invented to address something unaccounted for. So let's try.

- We collectively sense a phenomenon that needs to be addressed.
- Namely, there is a plurality of valid perspectives to account for.
- Translations between perspectives lead to responsive cohesion.
- We can solve big problems together if we can understand each other.

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- More precisely, CT is the theory of structure and translation.
- A perspective is a way of structuring reality.
- Taking on another's perspective (empathizing) is translating.
- Analogies preserve structure but change content.
- CT lets us design mathematical worlds and translations between them.

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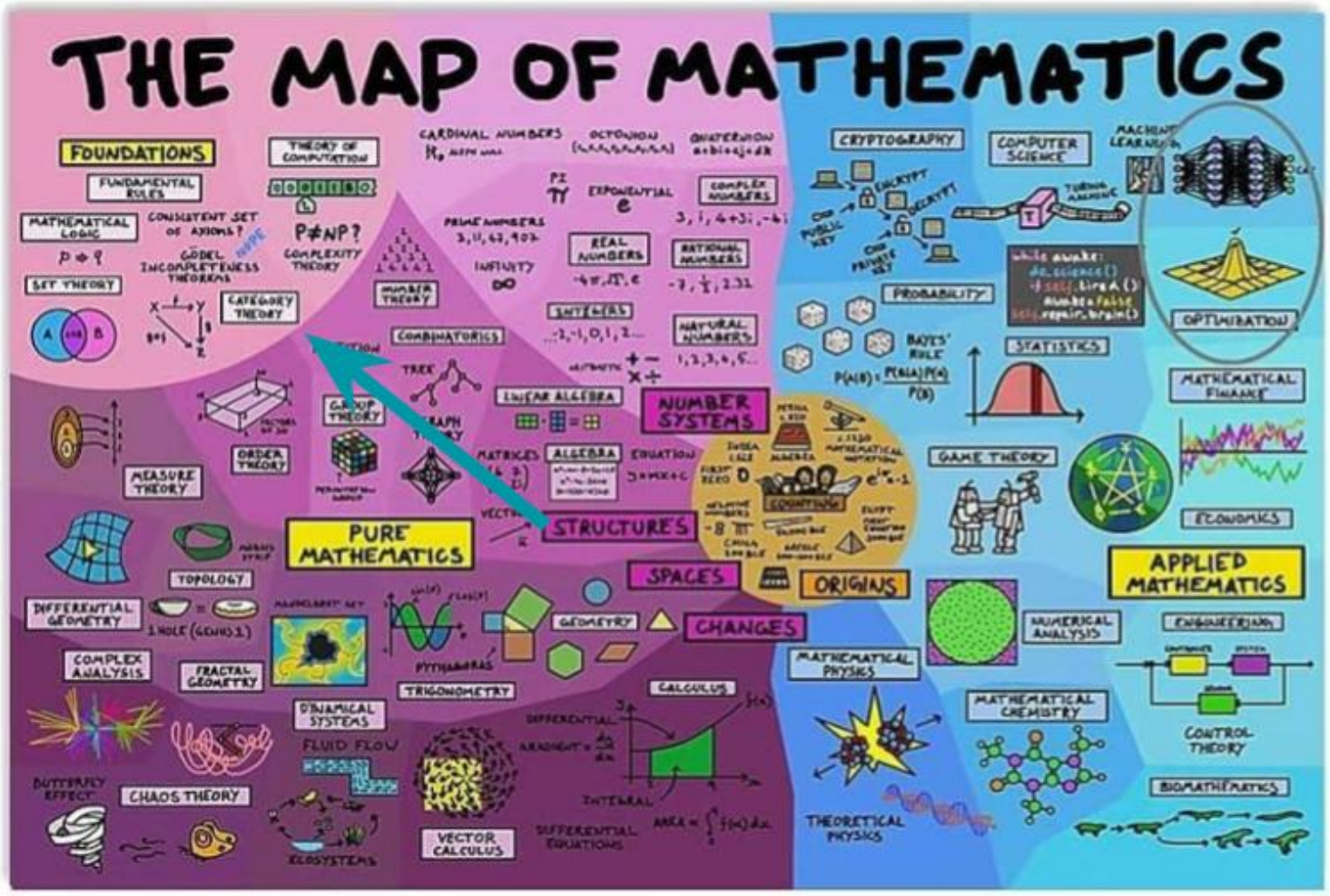
So does category theory do the job we're looking for?

- Suppose our goal is for beauty and love and understanding to flourish.
- We need something that approaches the pluralistic ideal, while still...
- ...being accountable enough that we can coordinate around it.

Outline

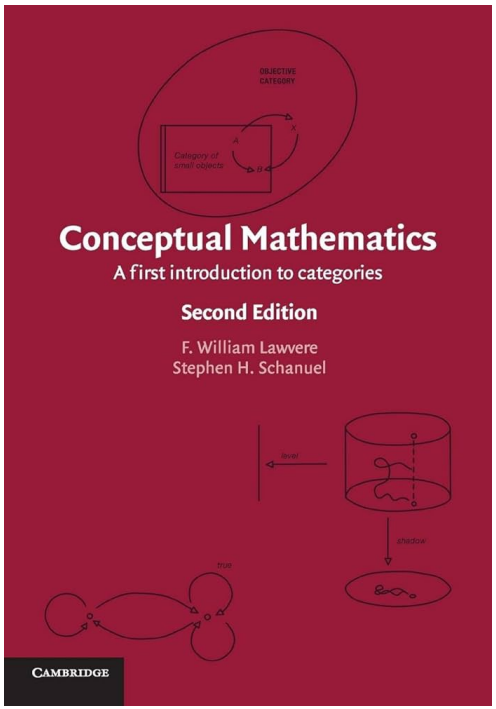
Perspectives on category theory (I)



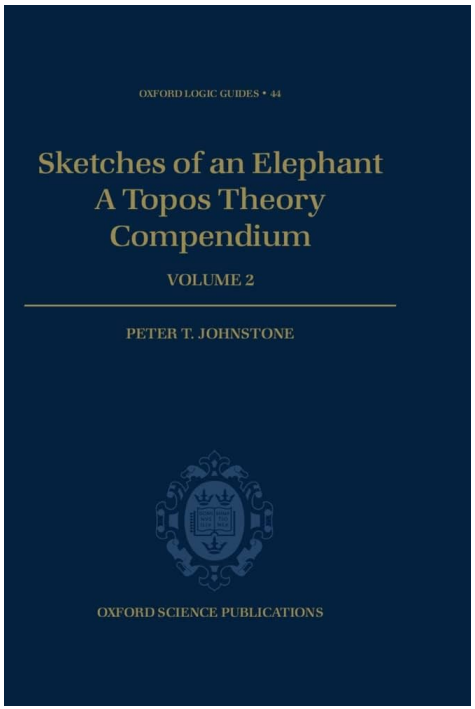
Denise Gaskins' Let's Play Math

Here it is in "foundations". It can serve in but is not limited to that role.

Perspectives on category theory (II)



Mathematistan, by Martin Kuppe



This is more like it.

- Category theory is mathematics for conceptualizing.
- It takes in the whole, understanding structure and seeing connections.
- It allows us to sketch perspectives on what we see, comparing results.

Moving toward accountability

That's all pretty squishy. Let's get more accountable.

- Category theory was invented in the 1940s by Eilenberg & Mac Lane.
- It was invented to account for a notion of "naturality".
- People had gotten used to translating between mathematical fields.
- They wanted to know the rules for translating between translations.
- Phenomenon of "natural transformation" needed theoretical backing.

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What they constructed was a theory of structure and structure-preservation.

- Algebra is structured. You can add, subtract, multiply, etc.
- Geometry is structured. You can draw continuous lines, curves, etc.
- Each of these can be made into a "category," a conceptual world.
- Categories organize thinking by specifying objects and relationships.
- A category is like a mathematical field. It is an object of study.
- Categories can be related by structure-preserving transl'ns: "functors".

Why are we here?

Heidegger said that our being is disclosed as care.

- If we don't care, we don't show up.
- So what do you care about here and now?
- We find ourselves in this room; let's start here again together.

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I'll use language to account for how we got here and where we're going.

- We are here to authentically ask about how math can promote life.
- I guessed that Plato and Aristotle would see sense-making as good.
- I proposed that accountability leads to collective sense-making.
- I cast mathematical fields as well-ruled accounting systems.
- I sought to account for our ability to translate between perspectives.
- I offered category theory (CT) as an appropriate accounting system.
- I described why CT was invented: to account for natural transform'ns.

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Category theory lets us create custom accounting systems and relate them.

- I claim it therefore makes us more accountable.
- It lets more of us be Newtons: explain our insights broadly.

Where are we going

The remainder of this lecture aims to account for dynamic arrangements.

- If we want to solve big problems together, we must coordinate.
- To coordinate, to work together, we must adjust to one another.
- What we say affects each other and how we arrange ourselves.
- Who you listen to, trust, ignore, follow, or fight is the arrangement.
- The arrangement is dynamic when it is allowed to change naturally.

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Try on the following perspective and see how the words feel to you.

- We seek a system where beauty and love and understanding flourish.
- This may not happen by chance; it requires collective sense-making.
- We aim to promote exchange among a plurality of diverse perspectives.
- Accountability—including accountable governance—is crucial.
- We want the arrangement to be dynamic, developing appropriately.

Outline

Boundary, interior, and umwelt

What do all things have in common?

- It depends perhaps on what you mean by thing.
- I mean: a person, a chair, a cow, a company, a cell.

Boundary, interior, and umwelt

What do all things have in common?

- It depends perhaps on what you mean by thing.
- I mean: a person, a chair, a cow, a company, a cell.

I propose that every thing has an *interface*, a boundary.

- There's a membrane through which it interacts with the world.
- It's all the ways you can *do* and all you can *receive*.
- Person etymologically means "to sound through"... this interface.

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A thing's interface is inhabited by the thing's interiority.

- I'm talking *at* your interface; I'm talking *to* you.
- You are a dynamic entity that inhabits your interface.
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On the other side of the thing's interface is its *umwelt*.

- This is the totality of what's inhabited outside the interface.
- It's your "universal other"; the character of what's exterior.

What is an interface?

So what exactly is an interface?

- Mathematically, we can account for interfaces as *polynomial functors*.
- This is some of the most elegant math I've ever seen.
- But that won't be helpful for our collective sense-making today.
- Come to my talk tomorrow in the CS department if you're interested.

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So how should we think about interfaces?

- Basically, an interface consists of possible outputs and inputs.
- What positions can you be in and in each, what can you receive?
- A chair outputs its position and rotation.
- It receives inputs (forces) through legs, back, etc.

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Note that what you can receive may depend on your current position.

- When your position has eyes closed, you receive less.
- When a folding chair is folded, it receives forces differently.
- When a protein is folded, it receives forces differently.
- When a hedgehog is curled in a ball, it receives differently.

Arranging interfaces in an interface

The simplest sort of interfaces are those that don't change.

This little thing is called a NAND gate.

- It's the most basic logical operation: all others are built from it.
- Its interface is: output a boolean (`true` or `false`)...
- ...and input two booleans. That interface never changes.
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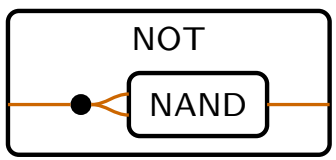
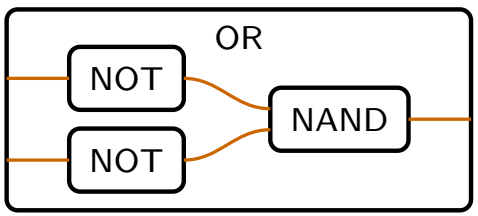
We build up other operations by arranging NAND gates together.





Similarly we build everything in your computer.

- All logic gates, adder circuits, ALUs, memory, CPUs, etc.

Arrangements as abstractions

This is getting very abstract and meta.

- We are talking about people and chairs and circuits in the same terms.
- When I say "interface", it's like an X variable to be filled in later.
- Once the variable is filled in, then all these terms have meaning.

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What is the point of abstraction?

- We take complex, low-level details and forget them for a moment.
- We look instead at the big picture, the high-level idea.
- This helps us decide what action to do, at least at the high-level.
- The last step is to translate the high-level action back to the detailed.
- Submarine: the dashboard provides an abstract view of what's outside.
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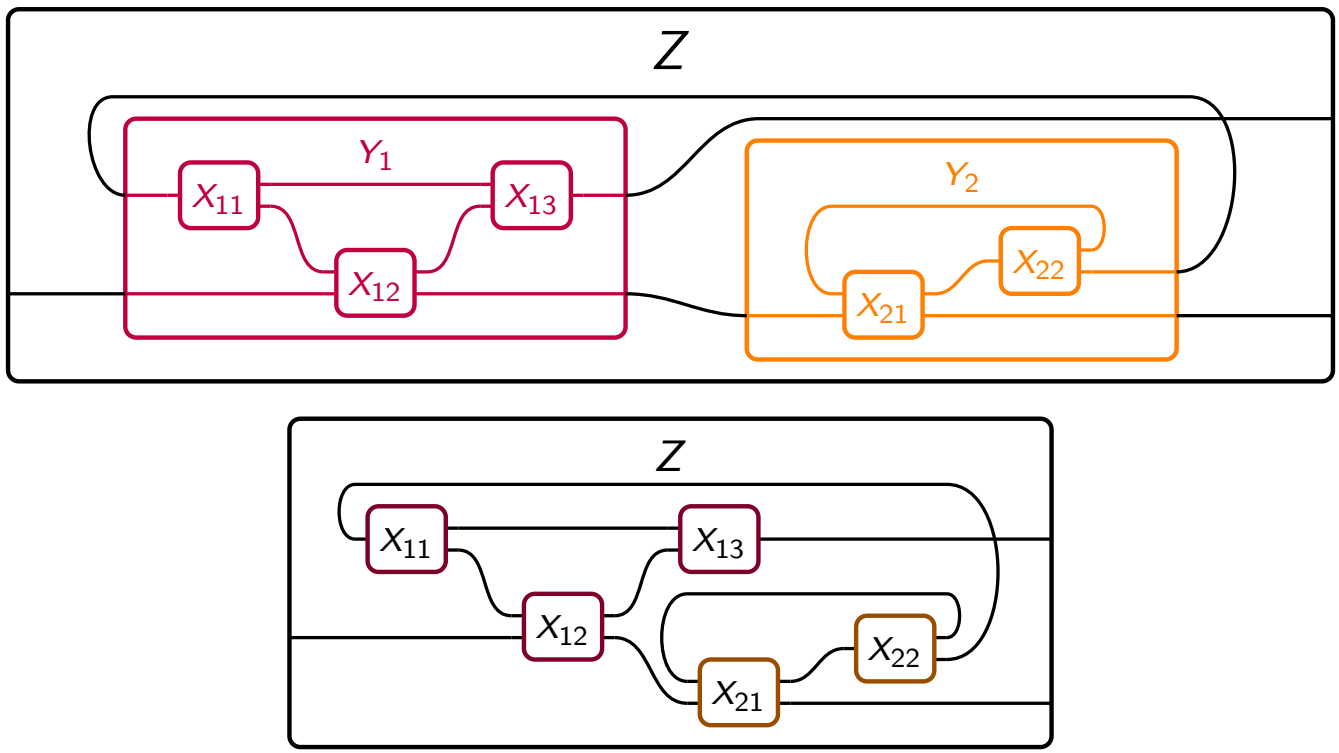
This notion of abstraction agrees perfectly with *natural transformation*.

- Polynomial functors are interfaces, natural transf'ns are abstractions.
- It's also what you get from the wiring diagrams shown previously.
- The outer interface abstracts away the details of the inner systems.
- But how it's used is passed inward and affects all those inner systems.

Nesting arrangements

We can nest arrangements within arrangements, arbitrarily deep.



Thus the theory of interfaces and abstractions is scale-free.

- The notion of interface works at any scale, from molecular to societal.
- The notion of arrangement is what forms higher level abstractions.
- Right now the arrangements are static, soon they'll be dynamic.

Dynamic arrangements

All the pictures so far have static arrangements.

- The circuitry is soldered in, unchanging for all time.
- But bodies and cells and brains and companies don't work that way.
- We're about to change how this very conversation is arranged.
- Rather than me talking and you listening, we'll move to Q&A.
- What's said in an arrangement (e.g. "thanks!") can cause it to change
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- Slime molds, mycelium netw'ks change based on what happens in them
- Companies change suppliers based on the interaction.
- Your brain's connectivity pattern changes based on what happens in it.
- Deep learning, prediction markets, Hebbian learning: all the same.
- The math of polynomial functors accounts for dynamic arrangements.

Recombinant intelligence

Computers are arrangements of transistors into higher abstractions.

- Build logic gates, latches, adder circuits, ALUs, all the way to CPUs.
- It's all static. The future will be dynamic.
- AIs, e.g. LLMs, are dynamic, but they're also monolithic.

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My bet for the future is what I might call "recombinant AI."

- Think of it like a game of coordinated fit-finding or problem solving.
- It will involve humans and LMs and machines and entities of all sorts.
- Come together, find our fit, make sense of a problem, and get to work.
- Once finished, the team disperses. In the process, each entity learns:
- Something about the problem, about themselves, and about others.

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Every collective asks itself this pun: "Do we make sense together?"

- Conjecture 1: We make sense together—we fit, cooperate, succeed—
- ...when we make sense together—produce sense, deepen understanding
- Conjecture 2: In order to make sense together, we need accountability.
- Conjecture 3: When arrangements are dynamic, the ability to give...
- ...and receive accounts naturally engenders amenability to feedback.

Outline

Summary

Our question is: "How can math promote life"?

- The ancients: Plato's "order / math" and Aristotle's "flourishing".
- Collective sense-making—orderly tracking—is central to both.
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Thanks!