Applied Category Theory:
Towards a hard science of interdisciplinarity

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Outline

Why am I here?

In 2007, I read *The Moment of Complexity* by Mark C. Taylor

- It explained that the world was getting increasingly complex.
- More would be different: Anomalies would become the norm.
- It presaged fake news, deep fake videos, difficulty knowing what's true.

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- My neuron pattern and yours are more different than our fingerprints.
- And these are quite different than the database schemas found in orgs.
- Despite these massive differences, we are able to communicate! How?
- For the world to navigate, we must communicate better.
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Navigation requires coordination. Our efforts need to be more coherent.

- We need to understand interdisciplinarity: how to learn from others.
- I want this to be a hard science, i.e. to be supported by math.

Accounting

We solve big problems together by coordinating our activity.

- When my efforts and yours conflict, it causes friction and loss.
- When we coordinate, we stop stepping on each others' toes.
- To work collectively, our activities must align. How do we align them?
- We give accounts. We explain our activity in *terms* of the collective.

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- Or if your account hides key variables, externalities that I must handle.
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Note: regularity is different than predictability.

- A chess game is regular (pawns don't move left), not predictable.
- Regulation: "Hey, you can't move a pawn left"; "Oh, oops!".

Mathematical fields as accounting systems

I think of mathematical fields as crystalized accounting systems.

- Arithmetic accounts for the flow of quantities, as in finance.
- Hilbert spaces account for the states of elementary particles, as in QM.
- Probability distributions account for likelihoods, as in game theory.
- Calculus accounts for relative rates of change.
- They're all crystalline in the sense of being structured and lawful.

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- Carefully track the phenomena, articulate the structure, systematize.
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Category theory (CT) is the accounting system for interlocking structures.

- Mathematical definitions are composed of interlocking structures.
- Category theory tracks the layers of structure and their connections.
- This makes analogies—similarities of structure—into formal objects.
- It accounts for the fact that different accounting systems cohere.
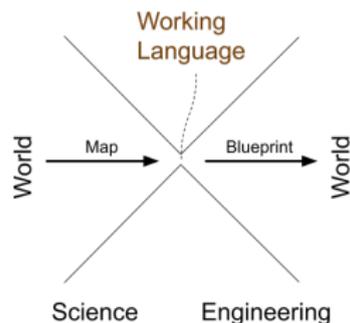
Science, math, and engineering
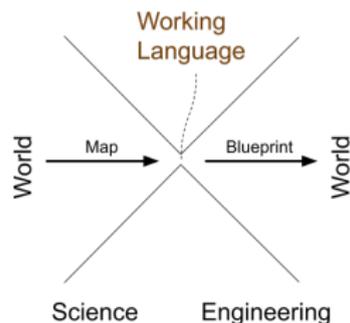
Roughly: science compresses, engin'ring elaborates.

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Category theory is like a conceptual stem cell.

- A stem cell can differentiate into huge variety of forms.
- Coming from a common origin, these forms work together coherently.
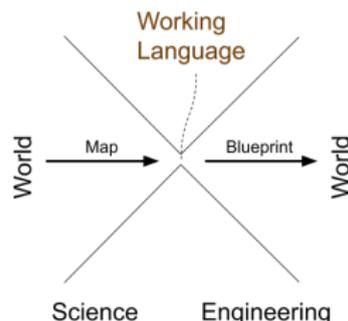
Category theory as conceptual stem-cell

Category theory (CT) can differentiate into many forms:

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Category theory as conceptual stem-cell

Category theory (CT) can differentiate into many forms:

- All forms of pure math... (algebra, topology, logic, number theory, differential equations...)
- Databases and knowledge representation (categories and functors)
- Functional programming languages (cartesian closed categories)
- Dynamical systems and fractals (operad-algebras, co-algebras)
- Shannon Entropy (operad of simplices, internal algebras)
- Taxonomies, metric spaces, and networks (enriched categories)
- Measurements of diversity in populations (magnitude of categories)
- Open economic game theory (Lens categories)
- Collaborative design (enriched categories and profunctors)
- Petri nets and chemical reaction networks (monoidal categories)
- Quantum processes and NLP (compact closed categories)
- Disease modeling and compartmental models (hypergraph categories)
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Popper's objection

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We counter this objection in two ways:

- Couldn't the same objection be made about mathematics?
    - Mathematics is the basis of hard science, used everywhere.
    - CT—like math—explains, models, formalizes many many things.
    - Conclude that math/CT explains everything and hence nothing?

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- Stem cells don't do work until they differentiate.
    - "Adult-level" work requires differentiation and optimization.
    - But the unified origins lead to impressive interoperability.
    - That's what we need for interdisciplinarity.

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- And it's branched out from math in a big way.
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- Operads:

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Let's focus on one.

- Operads: compositional arrangements of things within things.
- They're a branch of category theory that well-represents the spirit.

Plan of the talk

Overarching idea: CT as math to scaffold accounts from many disciplines.

- To pick one, we'll discuss *operads*: compositional arrangements.
- I'll sketch a definition and give a lot of examples.
- I'll explain dynamic operads, e.g. for learning and prediction markets.
- I'll conclude with a summary.

Outline

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Operations take arrangements of many sorts and produce one sort.

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- And ways to arrange them, $\varphi\colon X_1, \ldots, X_k \to Y$,
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Slightly more formal definition to come.

Operads are everywhere

Operads are used unconsciously in many fields.

- Electrical engineering: "wiring diagrams"
- Design: "set-based design"
- Computer programming: "data flow"
- Natural language processing: "grammars"
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We want to bring operads to the fore.

- There's a common theme in the way we think.
- Operads structure this sort of thinking.
- With mathematical structure, we can go much further.

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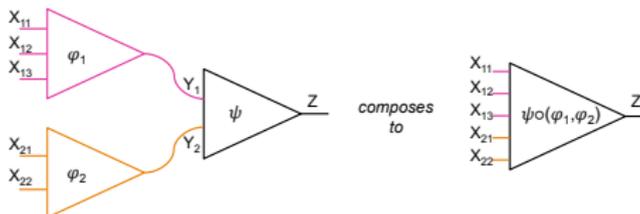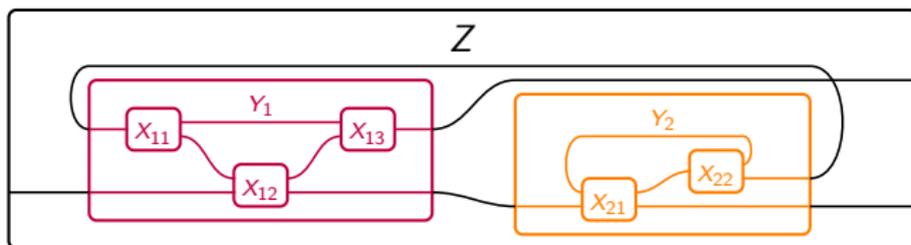- With mathematical structure, we can go much further.

Let's look for sorts, arrangements, and nesting in some examples.
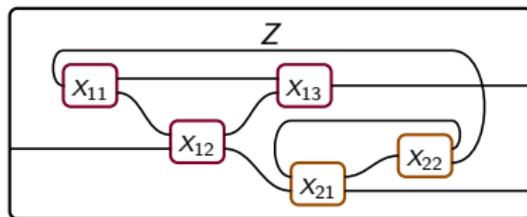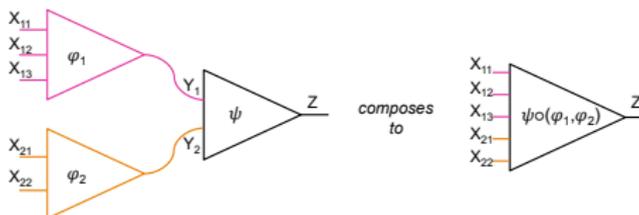
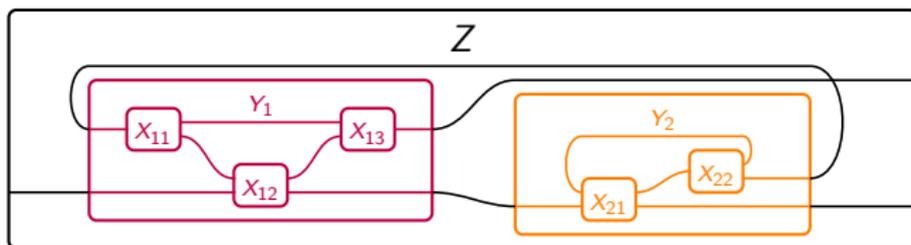# Operad 1: wiring diagrams
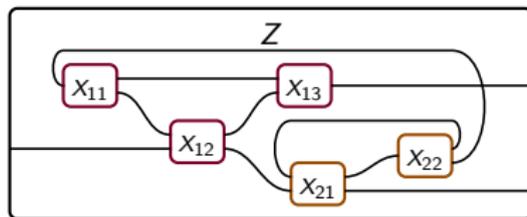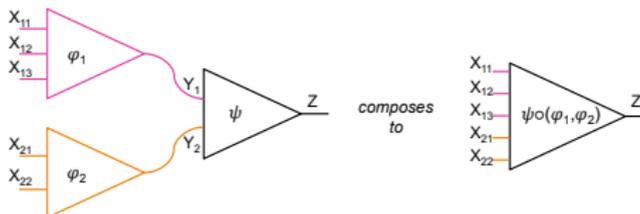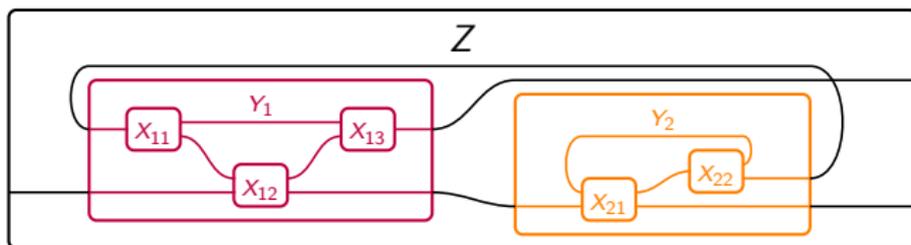
Sorts: boxes with ports.

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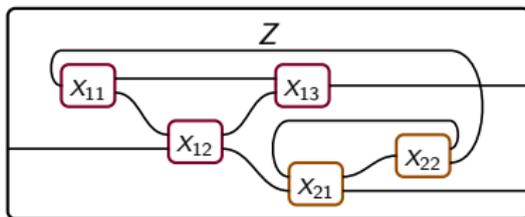
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Operad 1: wiring diagrams

Sorts: boxes with ports.   Arrangements: wiring diagrams.   Nesting: nesting.

Formal definition of operad

An operad $\mathcal{O}$ consists of

- A set $\mathrm{Ob}(\mathcal{O})$, elements of which are called *sorts*.
- For sorts $X_1, \ldots, X_k, Y \in \mathrm{Ob}(\mathcal{O})$, a set

$$\mathrm{Mor}_{\mathcal{O}}(X_1, \ldots, X_k; Y)$$

  Its elements are called *morphisms* or arrangements of $X_1, \ldots, X_k$ in Y.
  A *k*-ary arrangement $\varphi \in \mathrm{Mor}_{\mathcal{O}}(X_1, \ldots, X_k; Y)$ may be denoted

$$\varphi \colon (X_1, \ldots, X_k) \to Y.$$

- For each sort $X \in \mathrm{Ob}(\mathcal{O})$, an identity arrangement $\mathrm{id}_X \colon (X) \to X$.
- A composition, or nesting formula, e.g.,

$$\psi \circ (\varphi_1, \ldots, \varphi_k) \colon (X_{i,j}) \xrightarrow{\varphi_i} (Y_i) \xrightarrow{\psi} Z.$$

These are required to satisfy well-known "unital" and "associative" laws.

Operad 1: WDs again

An operad $\mathcal{W}$ for composing wiring diagrams:

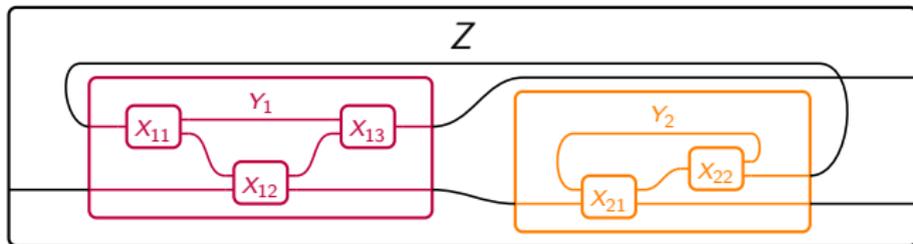- Sort $X \in \mathcal{W}$: any possible box-with-ports.

Operad 1: WDs again

An operad $\mathbb{W}$ for composing wiring diagrams:

- Sort $X \in \mathbb{W}$: any possible box-with-ports.

- Arrangement $\varphi \colon X_1, \ldots, X_k \to Y$ in $\mathbb{W}$: any wiring of X's in Y.
- Nesting: the facts about this fractal of wiring possibilities.

- (You could imagine an open dynamical system in each box.)

$\mathbb{W}$ is the decision of what sorts and arrangements you're considering.

Operad 2: hierarchical protein materials

There is an operad $\mathcal{M}$ for composing hierarchical protein materials.

- Why protein materials?
    - Protein materials include your skin: stretchable, breathable, waterproof.
    - Eat hamburgers, make amazing material.
    - Materials scientists would *love* to make materials like this.

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- A protein is an arrangement of simpler proteins.
    - There are "atomic" proteins: amino acids.
    - arrange in series or parallel (H-bonds), or
    - arrange in helices, double helices, any conceivable curve, etc.
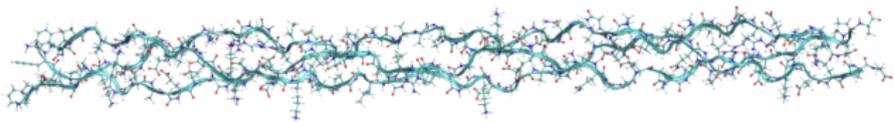
- Collagen has a nested structure: it is an array, each fiber of which is a triple helix, each strand of which is a helix, each unit of which is an amino acid.[1]

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Operad 3: probabilities

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Arrangement: "In this event, there's a distribution on next events."

- coin flip: $f = (\frac{1}{2}, \frac{1}{2}) \in \mathscr{P}_2$.
    - In the event coin flip, there's a 50-50 distribution on next events.
- die roll: $r = (\frac{1}{6}, \ldots, \frac{1}{6}) \in \mathscr{P}_6$.
- card selection: $p = (\frac{1}{52}, \ldots, \frac{1}{52}) \in \mathscr{P}_{52}$.

The nesting rule composes distributions by weighted sum:

- Flip a coin: result decides whether to roll a die or pick a card.
$$f \circ (r, p) = \left(\underbrace{\frac{1}{12}, \ldots, \frac{1}{12}}_{6 \text{ times}}, \underbrace{\frac{1}{104}, \ldots, \frac{1}{104}}_{52 \text{ times}} \right) \in \mathscr{P}_{58}$$

A zoo of operads: Grammars

Any context-free grammar is an operad.

⟨sentence⟩	::=	⟨noun-phrase⟩ ⟨verb-phrase⟩
⟨noun-phrase⟩	::=	⟨pronoun⟩ | ⟨proper-noun⟩ | ⟨determiner⟩ ⟨nominal⟩
⟨nominal⟩	::=	⟨noun⟩ | ⟨nominal⟩ ⟨noun⟩
⟨verb-phrase⟩	::=	⟨verb⟩ | ⟨verb⟩ ⟨noun-phrase⟩ | ⟨verb⟩ ⟨prep-phrase⟩
⟨prep-phrase⟩	::=	⟨preposition⟩ ⟨noun-phrase⟩

How is this an operad?

- The sorts are the parts of speech.
- The arrangements are the production rules.
- Nesting is nesting.

An arboretum of operads: Recipes

Each recipe is an operad.

- Combine sub-recipes to make a recipe.
- The outline for this talk is a recipe for getting an idea across.
  - Sorts: points I want to make.
  - Arrangements: putting points together to make a bigger point.
  - Nesting: the well-known outline structure.

Joey's Shakshuka (serves 6-8)

E. Eggs (2 per person)
On. Onion (1 big)
TP. Tomato Paste (4-6 oz)
TC. Canned tomatoes (56 oz)
Oo. Olive oil
F. Feta cheese
G. Cookable green (spinach, swiss chard, kale etc.)
V. Eggplant and/or other veggie
C. Cumin
(or) Parsley/Cilantro/Lemon
SP. Fresh serrano pepper
P. Pita
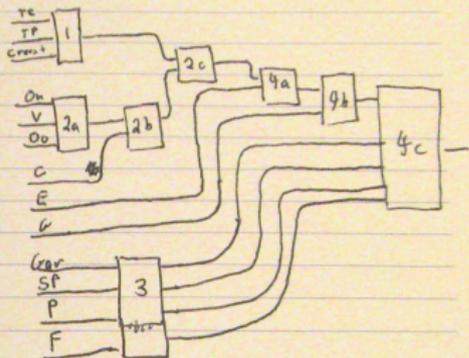
1. Tomato sauce: iF TC are whole, mash them. Add TP. Put in "Creuset" - casserole pan.

2. Sautee onion (On) and Veggies (V) in olive oil.[a] When almost cooked, add Cumin (C). Add to creuset. Simmer for ≥ 40 mins.

3. Prepare garnishes: cut parsley, cilantro, lemon (br), cut serrano pepper (SP) and serve along with Pita (P). Also

4. About ___ minutes before eating, add eggs (E) uncooked to creuset, when they'll poach. A few minutes later, add greens (G). Serve when cooked.

Summary on operads, and a quick word on algebras
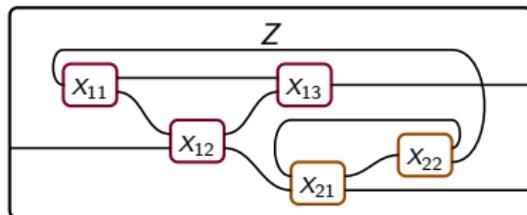
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- They let you specify how things in your arena of interest are built.

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- They let you specify how things in your arena of interest are built.

I didn't talk about algebras, but they're important too.

The operad is syntax, the algebra is semantics: something it can be about.

- Imagine putting a little machine into each of these little boxes.
- Any arrangement of machines makes a new machine.
- So wiring diagrams can be "about" machines.

Summary on operads, and a quick word on algebras

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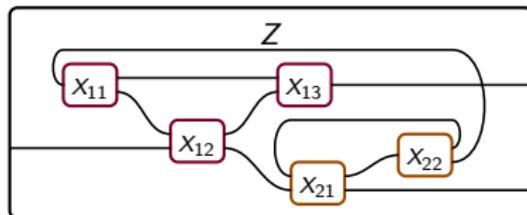The operad is syntax, the algebra is semantics: something it can be about.

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So far, all of our arrangements have been static. Let's relax that.

Outline

Even machines change over time

People often refer to functions as machines.

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- Your shoes wear down according to how you walk.
- Similarly for your baseball glove, your brain, your home.
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I want to call such a thing a *dynamic function $A \to B$*.

- They're modeled by *Mealy machines*, i.e...
- ...a set S of "states" and a function $f : S \times A \to B \times S$.
- For any state $s : S$, you get a machine that takes an $a : A$ and...
- ...not only gives out a B but also updates the state.
- The keys, shoes, glove, brain, and home respond to the input...
- ...and can also be changed by it.

Deep learning, prediction markets, organizations

We'll soon see how to similarly generalize operads. But first, applications.

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    - The state is the current weights and biases of the ANN.
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We (joint with B.T. Shapiro) have a math model of the first two.

- Also (joint with S. Libkind) a simplified model of Hebbian learning.
- These are all examples of a single mathematical structure.

Dynamic arrangements

Recall that operads were systems for nestable arrangements, e.g.:

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In each case, what would it mean for the arrangements to be dynamic?

- WDs that rewire based on what flows along the wires.

- Protein mat'ls who's configuration changes based on how it's pulled.
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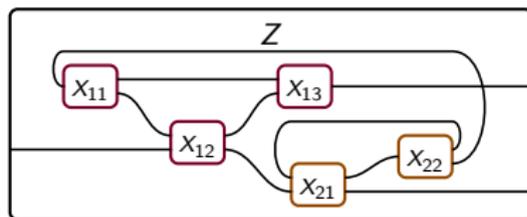- Grammatical constructions that change based on commun've success.
- And so on.

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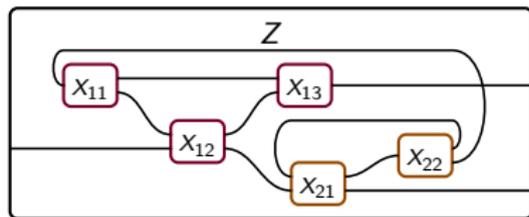- And so on.

The state controls the arrangement, and it changes based on what occurs.

Dynamic operads

A *dynamic operad*[2] is a coherent system of dynamic arrangements.

- The coherence means: the changing arrangements nest lawfully.
- Examples: deep learning, Hebbian learning, prediction markets,...

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I would love a dynamic operad of wiring diagrams, but I can't think of one.

- How we arrange themselves—who you hang out with—is v. important.
- And same at all levels: certain things like to work together.
- Shouldn't there be some coherent system for this?

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Interdisciplinarity is important and possible, yet not fully understood.

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Hopefully this gives a taste of ACT. It's really fun and increasingly useful.

Thanks! Comments and questions welcome...