Dynamic Interfaces and Arrangements: An algebraic framework for interacting systems

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2024 / 04 / 17

Outline

1 **Introduction**
- Applied category theory in living form
- Morphogenesis of healthy systems
- Today's talk

2 The current dynamic arrangement

3 Algebraic theory of interfaces and arrangements

4 Applications: Circuits, deep learning, and biology

5 Conclusion

Why am I here?

I'm here seeking a valuable exchange of ideas.

- Active inference offers a compelling but unusual worldview.
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- ...which I've gotten from applied category theory.
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Today I want to tell you about ACT, specifically in reference to interfaces.

- We interact through our interfaces (Markov blankets?).
- What sort of algebra could support us in thinking about this?

Mathematical fields as accounting systems

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Category theory is the accounting system for coherent structures.

- It makes analogies—similarities of structure—into formal objects.
- It's been useful in math, CS, physics, materials science, linguistics, etc
- What sort of system accounts for dynamic interaction?

Driving question: what do we have here?

What I want to account for is the incredible world we have.

- On earth we have amazing forms of life, from cells to humans.
- We have the built world, from transportation systems to computers.
- We have language and a systematic presentation of knowledge.
- We have morality, rules of thumb for living a good life.
- Each of these evolved through the push and pull and struggle of living.

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What are these systems and how do they develop?

- How can we talk cleanly about all these systems at once?
- What language is appropriate for giving accounts of it in action?
- Can we use the same language to engineer new systems?
- And what constitutes health, giving the system a sense of direction?

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The only part of this I'll discuss today is a potential accounting system.

This is the subject of the talk

My subject today is dynamic interaction.

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- Do you see how the math should be capable to describe *this*?

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We'll mostly ignore this self-reflective character, but it's hanging around.

Compressed theory and experimental feedback

I think compression and elaboration play a big role in the story.

- We compress our past into a form we can elaborate in a present.
- DNA compresses who died and who thrived into a language (ACGT).
- But theory is also a compression of past experience into language.
- Both are elaborated in the present, as I speak this theory here.
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Experiment and theory exist in both the biological and mathematical fields.

- Experimentation in math is attempts to articulate and compute.
- We value formalism F if it makes expression and computation *easy*.
- What is XIV * VI? $14 * 6 = (10 * 6) + (4 * 6) = 84.$ Ans: LXXXIV.
- Hindi-Arabic numerals are *empirically better* than Roman numerals.

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That's the sort of value claim I'm making about what I'll discuss today.

Plan for today's talk

The rest of today's talk will be in four parts:

- Give a presently-available case of what I'm trying to model.
- Whirlwind tour of what the math actually looks like.
- Talk about existing applications and open questions.
- Conclude with a summary.

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2 The current dynamic arrangement
- Summary of dynamic arrangements
- Interfaces
- Arrangements

3 Algebraic theory of interfaces and arrangements

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This zoom call as a dynamic arrangement

We're here on a call together. How can we begin thinking about this?

- Let's break it into three structures: interfaces, dynamics, interaction.
- Each one of us has an interface: what we can express and take in.
- Each one of us has dynamics: our internal state and how it updates.
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We draw boundaries around stuff, modularizing by nested reference frames.

- We have littler systems interacting within larger systems.
- This can be said of atoms and molecules, organizations and societies.

The math here will not be numerical: it will be *structural*.

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But before the math, let's make sure we understand our principal subject.

Our interfaces

The math should describe things we work with: like cells, tadpoles, and us.

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- But we need to ground this in something you can think about.
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So how do we think about ourselves here in this zoom call?

- Each of us can *do* certain things and *receive* certain things.
- What we can outwardly express and take in defines our *interface*.
- I'll call these *positions* and *forces*.
- Consider any expression, e.g. sound or attitude, as a kind of position.
- Your inner states are reflected outwardly as these positions.
- The world impinges on you, directs you, moves you by forces.

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What your body can receive in a moment depends on its position.

- When your eyes are open, your sensorium is bigger; more acts on you.
- When the car goes through a tunnel, the GPS stops receiving.

Our zoom arrangement and the enclosure

So here's the story so far: you output positions and receive forces.

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The way this works is based on the *arrangement* that we call zoom.

- This program arranges it so that we can input each others' outputs.
- Our interaction with the program may change the arrangement.
- E.g. spotlight, mute, etc., each changes how info is passed.

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1 Introduction

2 The current dynamic arrangement

3 Algebraic theory of interfaces and arrangements
- Interfaces as polynomials
- Arrangements and dynamics

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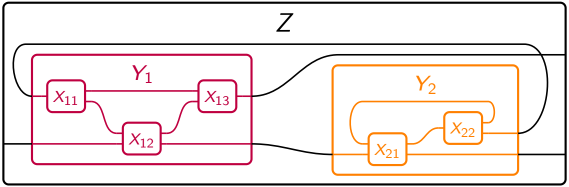
Recalling all the keywords we'll use

Interfaces, positions, forces, states, arrangements, enclosures, nesting.

- We each have an *interface*; it's that through which we interact.
- Our interface allows us to express ourselves through our *position*.
- Given a position, our interfaces allows us to receive certain *forces*.
- Our *state* is changed by the received force; it's expressed as position.
- Multiple interfaces interact together via their current *arrangement*.
- We also interact with our *enclosure* which is another interface.
- These arrangements can be *nested* hierarchically.



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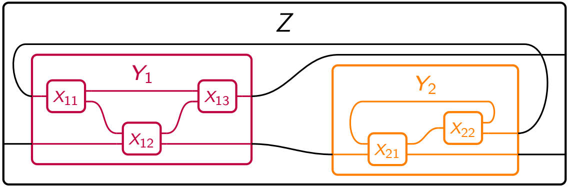

Arrangements can change through time based on what flows within them.

Interfaces as polynomials

As we said, an interface consists of two things.

- First, a set P of positions. Maybe $P = \{a, b, c\}$ or $P = \mathbb{R}^{44}$.
- Second, for each position $i \in P$, a set $F[i]$ of forces.
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We'll encode this as a polynomial in y with nonnegative integer coefficients.

- I know it's strange, but it works really well. It's a formal thing.
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- So imagine the interface is $y^5 + 62y^3 + 2y^0$. It has: ...
- ...1 pos'n with 5 possible inputs, 62 pos'ns with 3, and 2 with 0.
- What about $\mathbb{R}^3 y^{\mathbb{R}^{1,000,000}}$? It has $\mathbb{R}^3$ positions,...
- ...and in every one, its sensorium is $\mathbb{R}^{1,000,000}$.

Why polynomials?

So why do this craziness? Because the polynomial operations mean things.

- We have lots of operations: $p + q$, $p \times q$, $p \circ q$, $p \otimes q$, $p \vee q$, $[p, q]$.
- You've heard of the first three, but probably not the second three.
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Semantics of polynomial operations

Suppose p and q are polynomials representing interfaces. What's $p+q$?

- Well, $p+q$ is another polynomial, so it represents a new interface.
- Namely: that which can output a position of p or of q.
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- Example: $p = 3y^5 + 2y^4$ and $q = 6y^5 + y^3 + y^0$. What's $p+q$?

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What interface does the product $p \times q$ represent?

- It always outputs both a position $i \in p(1)$ and $j \in q(1)$, but...
- ...an input at (i, j) is either an input of p *or* of an input of q.
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All of the polynomial operations do something to interfaces.

- Composition $p \circ q$ runs p then q in series.
- Tensor $p \otimes q$ runs p and q in parallel.
- The Or-operation $p \vee q$ runs either p or q or both in parallel.
- The bracket $[p, q]$ runs arrangements for wiring p in q. Remember?

Arrangements

We can also formalize the notion of arrangement.

- Recall, we considered ourselves in this zoom call as an arrangement,...
- ...i.e., the way zoom lets my outputs be your inputs, and vice versa.
- Or think about cell-organs arranged in a cell or cells in a tissue.
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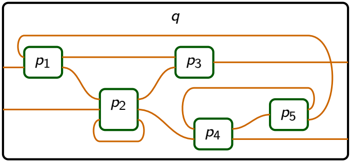

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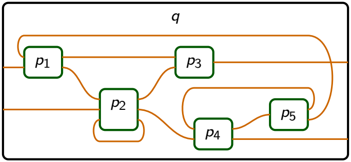

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Formally, an arrangement is a *natural transformation of polynomials*.

- First take all the little interfaces $p_1, \ldots, p_5$ and tensor them.
- An arrangement is a natural transformation $p_1 \otimes \cdots \otimes p_5 \to q$.

What's happening in this talk

Brief interlude, for meta-stuff.

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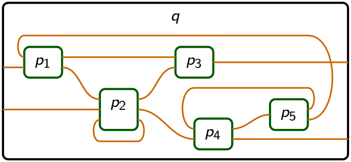
It's about interfaces: how you manipulate them, arrange them, nest them.

- Interfaces have outputs, and inputs that can depend on them.
- They're captured as polynomials, not as functions, but as structure.
- Then we can $+, \times, \circ$... these poly's to make new interfaces from old.
- And arrangements (and dynamics next) come with the math too.

Dynamics

Look at the arrangement again:



If each p_i had a dynamical system in it, then so would q.

- A dynamical system is a thing with states that evolve through time.
- It can be a system of ODEs, or just a high-level idea.
- You're in a state; this shows up as your position (or output).
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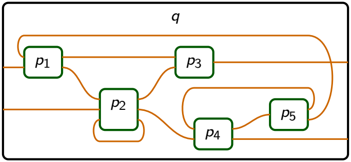

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Mathematically, a dynamical system on p is $Sy^S \to p$.

- Why am I telling you this?! Because all the math looks the same.
- Both arrangements and dynamics are *natural transformations*.
- We *compose* them to get q's dynamics from the p_is'.

Dynamic arrangements

This is the last math slide. Let's think about dynamics a bit more.

- If you remember a few slides ago, I was talking about operations.
- A dynamical system on $p + q$ can switch between p-mode and q-mode.
- A dyn'l system on $p \times q$ outputs both, receives from either.
- A dyn'l system on $p \otimes q$ outputs both, receives from both.
- A dyn'l system on $p \circ q$ does a serial protocol.
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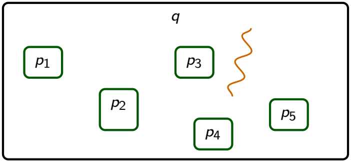
- What about $[p, q]$? We said it "runs arrangements for p in q".

In other words the operation $[-, -]$ is an interface for arrangements.

- A dynamical system on $[p_1 \otimes \cdots \otimes p_5, q]$ outputs arrangements...
- ...and receives whatever flows within the system.



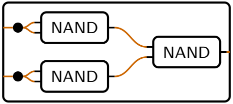
Outline

1. Introduction
2. The current dynamic arrangement
3. Algebraic theory of interfaces and arrangements
4. **Applications: Circuits, deep learning, and biology**
   - Circuits, control systems, and deep learning
   - Dynamic organizational structures
   - Application to active inference
5. Conclusion

Digital circuits and control systems

Digital circuits and control systems fit neatly into this formalism.

- A computer is a nested arrangement of dynamical systems.
- Two transistors make up a NAND gate.
- You can get an OR gate by wiring together three NAND gates.



The arrangement and dynamics are very simple.

- The arrangement is fixed, unchanging, soldered in.
- The dynamics are simple: state is a function of input only.
- But with enough nesting, you get something amazing: a computer.

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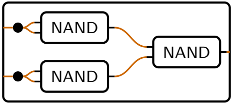

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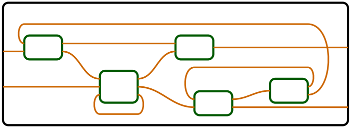
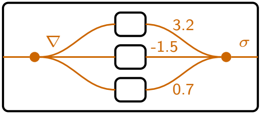
Control systems have complicated dynamics but fixed arrangement.



Deep learning

Deep learning also fits into this formalism in a different way.

- The interfaces are all the same: $\mathbb{R}y^{\mathbb{R}}$, outputting and inputting $\mathbb{R}$.
- And the arrangements are all very simple: activated weighted sums.



- The info flowing out of the inner boxes is the current "guesses".
- The current weighted sum is calculated and sent out as current guess.
- The info flowing into the big box is the "loss".
- It is distributed to the inner boxes according to the gradient.

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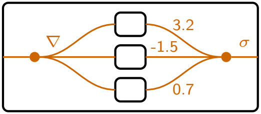

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But here the arrangement is *dynamic*!

- The loss coming in is not only sent to the little boxes,...
- ...it also updates the arrangement, the collection of weights, itself.

Dynamic organizational structures

Deep neural networks is one of four examples of a certain structure.

- B. Shapiro and I call it a *dynamic organizational structure*.
- It is a fractal-like system of arrangements that change through time.
- The arrangement decides how information flows through the system.
- And yet the flowing information can change the arrangement.
- We formalize this using category theory.

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I would guess that active inference can somehow be another one.

Articulate language to say... what?

Many people think of math as about number, but it's not.

- Each math subject is an accounting system to track certain things.
- What the work above lets you track is: dynamic arrangements.
- The math called *polynomial functors* is well-known and beloved.
- It beautifully accounts for interfaces, dynamics, and arrangements.
- It's about building up bigger systems from littler parts.
- It accounts for composing circuits&control and deep learning systems.
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But I think this has particular value in active inference.

- It lets us talk coherently and precisely about these things:
- ...changing interfaces, dynamics, how things communicate,...
- ...and how that communication pattern changes as info is exchanged.

Active inference as dynamic organizational structure

The main work to make active inference into an example of a DOS[1] is to:

- specify all the interfaces we will use to house free energy minimizers,
- consider how free energy minimizers arrange themselves, and
- say how these arrangements constitute a larger-scale FEM.

[1] One person suggested we use the term "Multi-Scale Dynamic Organizational Structure" in order to clarify our particular brand.

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As long as all this coheres nicely (in a formal way), we'll have a DOS.

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Applied category theory is math for tracking interlocking structures.

- It's not about how much there is, it's about how it's arranged.
- It applies in QM, CS, math, materials science, linguistics, physics...
- ...but it focuses on the structural questions in each one.

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I'm interested in the structure of interacting dynamical systems.

- I want to account for how we interact, right here on zoom.
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I don't have specifics, but I have the accounting system.

- The cat'y of polynomial functors accounts for dynamic arrangements.
- I think it may be helpful for thinking about active inference.

Thanks! Comments and questions welcome...