

An attempt to explain category theory to biologists in 15 minutes

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NITMB: Expanding the Palette of Mathematics in Biology
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Outline

1 What is category theory?

- Category theory is the math of math
- CT as conceptual stem cell

2 Operads

3 Conclusion

Category theory is the math of math

Mathematical fields are ways to account for various repeatable phenomena.

- Arithmetic accounts for the flow of quantities, as in finance.
- Hilbert spaces account for the states of elementary particles, as in QM.
- Probability distributions account for likelihoods, as in game theory.
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Category theory does this same thing for math.

- Invented in the 40s to form'ze a common but undefined notion: *natural*
- It connected disparate areas of math; used to prove major conjectures.
- It finds and distills the repeated phenomena from throughout math.

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Category theory does this same thing for math.

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The main phenom'n CT distills and formalizes: the power of analogy.

- Analogy is similarity of structure across difference of content.
- Unifying repeated phenomena across math lets you transport theorems.

Category theory as conceptual stem-cell

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- All forms of pure math... (algebra, topology, logic, number theory, differential equations...)
- Databases and knowledge representation (categories and functors)
- Functional programming languages (cartesian closed categories)
- Dynamical systems and fractals (operad-algebras, co-algebras)
- Shannon Entropy (operad of simplices, internal algebras)
- Taxonomies, metric spaces, and networks (enriched categories)
- Measurements of diversity in populations (magnitude of categories)
- Open economic game theory (Lens categories)
- Collaborative design (enriched categories and profunctors)
- Petri nets and chemical reaction networks (monoidal categories)
- Quantum processes and NLP (compact closed categories)
- Disease modeling and compartmental models (hypergraph categories)
- Deep learning and prediction markets (Dynamic monoidal categories)

Coming from a common origin, these forms work together coherently.

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- Couldn't the same objection be made about mathematics?
 - Mathematics is the basis of hard science, used everywhere.
 - CT—like math—explains, models, formalizes many many things.
 - Conclude that math/CT explains everything and hence nothing?

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 - Conclude that math/CT explains everything and hence nothing?
- Stem cells don't do work until they differentiate.
 - “Adult-level” work requires differentiation and optimization.
 - But the unified origins lead to impressive interoperability.
 - That's what we need for interdisciplinarity.

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 - This isn't mere analogy, it's analogy made rigorous.
- It's revolutionized pure math since its inception in 1940s.
 - It's touched or greatly influenced all corners of mathematics.
 - It's become a gateway to learning mathematics.
- And it's branched out from math in a big way.
 - Databases and knowledge representation ([categories and functors](#))
 - Functional programming languages ([cartesian closed categories](#))
 - Dynamical systems and fractals ([operad-algebras, co-algebras](#))
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1 What is category theory?

2 **Operads**

- Operads
- Examples
- Algebras

3 Conclusion

Operads

I'll tell you one branch of category theory, called *operads*.

- This is a framework for formal worlds of interfaces and operations.
- Danger: this is one small part of CT. Since it's all you'll know,...
- ...you may implic'y assume it's all of CT. "Biology \neq genetics!"

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An operad includes: **interfaces** and **arrangements** that **nest**.

- Tell me all the **interfaces** (types of thing) you'll be considering.
- Tell me how you let me **arrange** some interfaces inside an interface.
- Show how **nested** arrangements also count as arrangements.

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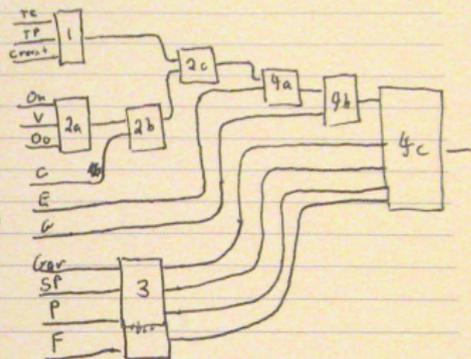
Operad 1: recipes

- **Interface** = "step", e.g. "prepare veggies" or "do carrots".
- **Arrangement** = some steps that constitute a (big) step.
 - prepare veggies = do carrots & roast brussels
 - do carrots = wash carrots & peel carrots & chop carrots.
- These **nest**: prepare veggies = WC & PC & CC & RB

Recipe for shakshouka

Joey's Shakshouka (serves 6-8)

- E. Eggs (2 per person)
- On. Onion (1 big)
- TP. Tomato Paste (4-6 oz)
- TC. Canned tomatoes (56 oz)
- Oo. Olive oil
- F. Feta cheese
- G. Cookable green (spinach, swiss chard, etc.)
- V. Eggplant and/or other veggie
- C. Cumin
- Gar. Parsley/Cilantro/Lemon
- SP Fresh serrano pepper
- P Pita



1. Tomato sauce: if TC are whole, mash them. Add TP. Put in "Creuset" - casserole pan.
2. Sauté onion (On) and Veggies (V) in olive oil (Oo). When almost cooked, add Cumin (C). Add to creuset. Simmer for ≥ 40 mins.
3. Prepare garnishes: cut parsley, cilantro, lemon (Gar), cut serrano pepper (SP), and serve along with Pita (P) and also
4. About minutes before eating, add eggs (E) uncooked to creuset, when they'll poach. A few minutes later, add greens (G). Serve when cooked.

Operad 2: hierarchical protein materials

There is an operad \mathcal{M} for composing hierarchical protein materials.

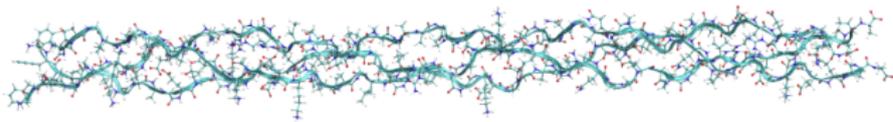
- Why protein materials?
 - Protein materials include your skin: stretchable, breathable, waterproof.
 - Eat hamburgers, make amazing material.
 - Materials scientists would *love* to make materials like this.

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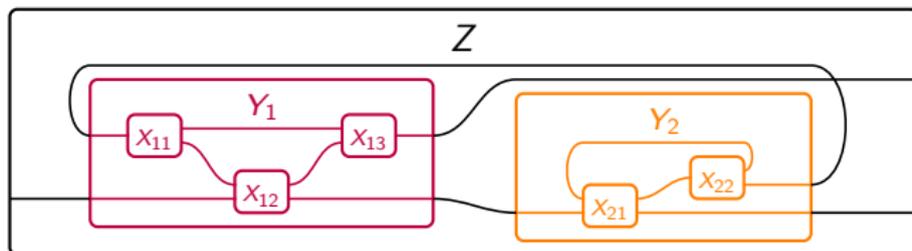
- Why protein materials?
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 - Materials scientists would *love* to make materials like this.
- A **protein** is an **arrangement** of simpler **proteins**.
 - There are “atomic” proteins: amino acids.
 - arrange in series or parallel (H-bonds), or
 - arrange in helices, double helices, any conceivable curve, etc.



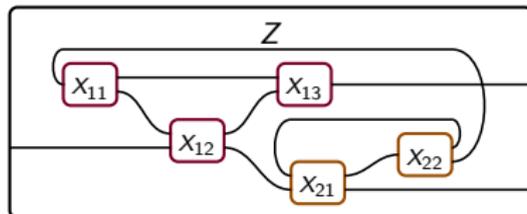
- Collagen has a **nested** structure: it is an array, each fiber of which is a triple helix, each strand of which is a helix, each unit of which is an amino acid.¹

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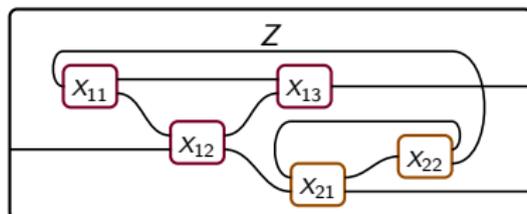
Operad 3: wiring diagrams



\Downarrow nesting



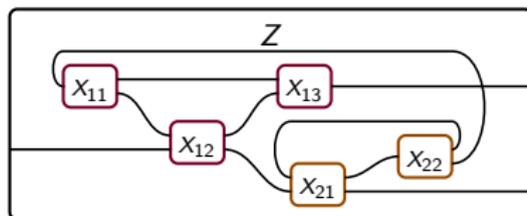
Algebras assign meaning



An operad describes what you can do; an algebra lets you do it.

- In the above case, it tells you “what can go in the boxes”.
- Example 1: Partially observable Markov processes
- Example 2: Continuous dynamical systems with inputs and outputs
- Example 3: Tensors (more-than-2-dimensional matrices).

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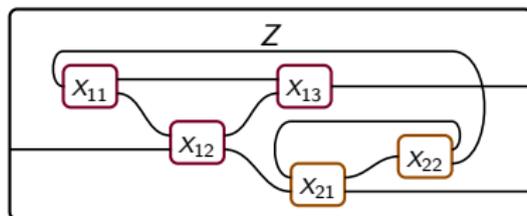
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Let's look at continuous DS's. Inside a box like: $a \text{---} \square \text{---} b \dots$

- ...put any system of ODEs $\dot{x} := f(x, a)$ and readout $b := g(x)$.
- A WD gives a formula for constructing new ODE systems from old.

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But there's also an algebra of tensors: put tensors in the boxes.

- The WD gives a formula for constructing new tensors from old.
- In fact “fixed points” constitutes a *natural* map from ODEs to tensors.

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Recommended references:

- Pure math: Leinster *Basic Category Theory*.
- Applications: Fong & Spivak *Invitation to applied category theory*.
- Philosophy and vibes: Cheng *Joy of Abstraction* and *How to bake π* .

Thank you for attending! Comments and questions welcome...