Accounting for our inventiveness

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TOPOS INSTITUTE

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Outline

Remembering how and why are we here

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- To care about, attend to (*memor*).

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Explain how energy got routed—in bare physics—to bring CTists to NITMB

What can we count on?

Colloquially, to count on something means to rely on it.

- The word “count” is a math word, so that’s a pun.
- In this talk, you can count on puns, etymology, and math.
  - I’m accountable for math claims being accurate.
  - Puns in this talk are deliberate: all readings being intended.
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We can count on what’s actual and what’s been tested.

- Actuality is self-evident: what’s here now. W/o it, testing is impossible
- In partic’lar, any claim about “our inventiveness” should hold for yours.
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We can count on mathematics for consistent accounting.

- Math is exceptionally self-consistent; it stands the test of time.
- It provides consistent accounting systems for phenomena.
- It's true like a wheel: a simple ideal serves as an attractor.

High-precision energy routing

High-precision energy routing has been increasing (roughly) monotonically.

- 4bya there were tides, tectonic shifts, etc. Energy went anywhere.
- Cells can swim, move energy around within themselves.
- Rabbits can direct energy to their limbs; humans get oil out of earth.
- Computers have trillions of voltages, routed with very high precision.

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Living things seem to recursively ||: route energy through switches that :||

- Define balance as *poise for symmetry breaking*.
  - Rest = pencil on its side; balance = pencil on its point.
  - A ball atop a hill, a joystick with all directions equally pressable.
- Switches are balance points: big changes from small nudges.
- Life seems to route energy through switches. When to switch?
- Usually: switch so that energy is routed through more switches.

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Increasing HPER seems to be a consistent phenomenon.

- Maybe "the cosmos knowing itself" (Sagan) has also been increasing.
- The point is that we might find a sense of direction from such $\nearrow$'s.

Inventiveness, portability, compositionality

I'm going to claim that humans are not the first to be inventive.

- For some this is obvious, for others it is heresy.
- Humans are very inventive so by IVT, it started somewhere. With us?
- To me, the standard dogma suffers from serious anthropocentrism.
- I'll give a (very idiosyncratic) account of inventiveness in biology.

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Portability enables compositionality, which increases HPER.

- Portability $\approx$ abstraction: same structure, new substrate.
- As forms get more portable, they can compose into larger collectives.
- Larger collectives route energy through more switches, more precisely.
- Repeatedly, biology has invented ways to make form more portable.
- I'll track portability as the mechanism; HPER is what it produces.

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How can we account for biology's ability to increase portability?

Mathematical fields as accounting systems

I think of mathematical fields as crystalized accounting systems.

- Arithmetic accounts for the flow of quantities, as in finance.
- Hilbert spaces account for the states of elementary particles, as in QM.
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We obtain a conceptual overlay: the phenomena are abstracted.

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- Set up a system: goat leaves/returns, move a rock back/forth.
- Given two fields of goats sep'd by a fence, $+$ models "remove fence".

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Category theory (CT) accounts for perspective-taking and analogizing.

- More precisely, CT is the theory of structure and translation.
- A perspective is a way of structuring what's there.
- Migrating from one perspective to another is translating.
- Analogies translate while preserving the structure we're tracking.
- CT lets us design mathematical worlds and translations between them.

Plan of the talk

- I'll give a very brief history of inventiveness in biology.
- I'll discuss how accounting inventions support portability.
- I'll discuss some math that's pretty and relevant.
- I'll conclude with a summary.

Outline

Lightning, hurricanes, and life

Eric Smith's 2007 talk, "Inevitable Life?", lays out a theory for life's origin.

- It is a *metabolism first*, rather than control (RNA) first, theory.
- It is a hell-theory (hot depths) rather than heavens-theory (sun god).
- The theory is formulated around the reverse TCA cycle.
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Smith likens early life to lightning strikes or hurricanes.

- With lightning, there's a potential difference between sky and ground.
- Lightning actualizes that potential by creating a spark.
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Smith says that early life was another such case.

- Deep sea hydroth'mal vents emit CH_4 and CO_2, high-energy molecules.
- There's a energy-emitting reaction $CH_4 + CO_2 \rightarrow 2CO + 2H_2$.
- Life was the simplest "chemistry lab" that could catalyze this reaction.

Abiogenesis: From physics to FUCA to LUCA

Portability is the abstraction I'll name that recurs over and over.

- White smokers provide containment: little capsules in rock.
- The first univ'l common ancestor (FUCA) encapsulated itself and left.
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Between FUCA and LUCA, a lot of great inventions got online.

- Proteins are molecular machines resp'ible for everything organisms do.
  - They're completely made from amino acids.
  - Each AA has a left & right side—forming chains—and data $SC : 20$.
  - The data is matter with properties: big?, charged?, hydrophobic?
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- The genetic code is a func'n: codons $64 \cong \{A, C, G, T\}^3 \to 20$ to AAs.
  - Out of 20^{64} func'ns LUCA found 99.9999%ile best for error corr'n.
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  - The genetic code is a sol'n to an info'n theory problem.
  - The search started with no genetic capacity, and bootstrapped.
- Invented repair: proteins that fix the protein-production machinery.

Bacterial machinery

By the time of LUCA, life had invented remarkable molecular machines.

- ATP synthase: an actual turbine that spins to produce ATP.
- ATP is a rechargeable battery: stores energy, releases it on demand.
- Flagellar motor: a rotary engine that spins a propeller.
- These are nanoscale machines with moving parts, built from proteins.
- Each is a catalyst coordinating large-scale activity, breaking symmetry.

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But how was all this invented?

- Turbines, batteries, motors, circuits: designed w/o a designer.
- Standard answer: random mutation + natural sel'n. Very slow & blind.
- But blind search doesn't build turbines. S'thing structured the search.
- This structure is what we're seeking, to account for our inventiveness.

The avalanche sculpts the ground

Per Bak (1987) introduced self-organized criticality (SOC) via a sandpile.

- The pile self-tunes to critical slope; no outside agent loads it.
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- In cells, stress triggers a response that rewires future sensitivity.
- In brains, failed predictions trigger rewiring: old accounting didn't fit.
- In societies, after a crisis cascades, institutions reorganize around it.

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One source of our inventiveness: the flow reshapes the ground.

- At every scale—molecular, cellular, neural, social—the same pattern.
- Cells built from mol. switches, brains from cells, societies from people.
- At each scale, the wiring can change based on what flows through it.
- Each level of portable coordination lays the groundwork for the next.
- The math of today is based on the math (and biology) of yesterday.

Back to the history: how did these scales get built from each other?

Eukaryotes: biology builds a city

An archaeon that needed H_2 engulfed a bacterium that made it.

- Not predation: archaea's flexible membrane wrapped around a partner.
- The bacteria mult'ied inside: a eukaryotic cell has many mitochondria.
- The membrane kept compartmentalizing to form ER, nucleus, etc.

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A Euk. is like a city with bacteria as citizens. But a city needs addresses.

- ER attaches sugar tags to proteins. These are address labels.
- The cell reads the label and routes the protein to the right room.
- Before: everything near where it was used, travels by diffusion.
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The city also has an external "public-facing" interface.

- Some proteins, *receptor proteins*, get routed to the cell surface.
- They're how the cell presents itself to and receives signals from outside.
- The cell changes which receptors it displays depending on its state.
- This is a *changeable interface*: the set of ports isn't fixed.
- Multicell'ity becomes poss. when cells can read each other's surfaces.

Mate selection: using more intelligence

Before sexual selection, fitness meant surviving to reproduce.

- The environment was a feedback loop from genome to genome.
- Sex began as DNA repair: import a matching sequence to fix a break.
- Repair, then deliberate shuffling, then mate choice: *smarter* filtering.

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With mate selection, an organism evaluates another organism.

- A chooser must navigate the lifeworld: find food, read threats, assess.
- Others can *see* not just if, but how, you survive. No longer blind.
- All the intelligence that has been created so far now contributes.

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This is an invention that improves our inventiveness.

- The filter itself now has intelligence; it's not just replication rates.
- A chooser models another organism: see and amplify what's working.
- More refined choice in the filter means faster, better-directed search.
- This invention accelerates directed genomic change, i.e. inventiveness.

From cells to this slide: many big inventions

Multicellularity: cells reading each other's surfaces, coordinating.

- Gap junctions: direct channels between cells, sharing ions and signals.
- Neurons: gap junctions specialized for speed and range.
- These allow fast feedback loops: outpace the environment to predict it.

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Written vs. "verbal" language. Distinction: can it exist without energy?

- Bodily movements, vocal chords: "verbal" process, requires energy.
- DNA, cave paintings, flash drives: "nounal", persistent without energy.
- Written language invented twice: DNA and $\sim$50k years ago.
- Turing systematized a relation between written language and process.
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We're still inventing ways to route energy more precisely and efficiently.

- The inventiveness goes back to before LUCA.
- Immense inventiveness. How do we account for this?

Words are cheap; we need to firm up our accounting.

Outline

The portability pattern

Each major invention in our story made form more portable.

- FUCA encapsulated itself and left the hydroth'l vent: a portable cell.
- The genetic code is a portable recipe, same form in new material.
- Sugar tags route proteins to the right address; transport upgrade.
- Written language: persists without energy, making plans portable.

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Portability enables composition into larger collectives.

- You can join more wholes, live in more places, handle more situations.
- A collective catalyzes at its scale: arranging to make outcomes happen.
- A larger collective has more choice points, more capacity for HPER.
- But parts may frustrate each other: your move causes me problems.
- A collective whose parts conflict wastes energy and won't persist.

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So coordination is key. What makes coordination at scale possible?

Accounting

To account: *a* (toward?) + *computare* (to reckon together).

- To bring a reckoning together. But toward what?
- Toward settlement: when all is accounted for, nothing is left over.
- An account is what you give so others can coordinate with you.

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A collective catalyzes at its scale. Accounting is what enables this.

- Parts that can't account for their activity can't coordinate.
- Accounting makes internal activity legible and regulatable.
- Regularity $\neq$ predictability. Chess is regular, not predictable.
- Regulation means correction: "you can't move a pawn left"; "oops!".
- Portability creates collectives; accounting holds them together.

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Math'l fields are crystalized accounting systems (arithmetic, probability,...).

- We want an account of the pattern we've been tracking:
  - Form becoming portable, allowing it to compose into collectives,
  - Collectives needing accounting in order to coordinate, and
  - Interaction patterns keep changing, based on what occurs there.
- Are humans the first to produce accounting systems?

Biology's first accounting systems

Recall: accounting makes a collective's activity legible and regulatable.

- An account is what you give so others can coordinate with you.
- When all is accounted for, when we find new balance: things settle.

Biology's first accounting systems

Recall: accounting makes a collective's activity legible and regulatable.

- An account is what you give so others can coordinate with you.
- When all is accounted for, when we find new balance: things settle.

DNA accounts for itself: it encodes what's needed to make DNA work.

- It encodes what the code is made of: nucleotides,
- ...how to copy it: DNA polymerase,
- ...how to read it and build proteins: mRNA, tRNA, and ribosomes,
- ...how to fix errors to maintain the code's integrity: repair enzymes.
- Nothing critical is left out: DNA is a Quine written in physics.

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- Signaling molecules are accounts: each cell reports its state to others.
- Gene regulation adjusts activity so the collective re-settles.
- Each new collective needs accounting to handle novelty at that scale.
- Biology invented accounting 3.5 billion yrs before humans did.

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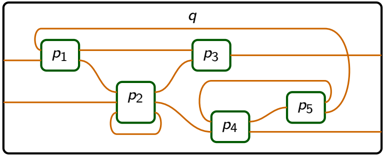
Next, we'll try (mostly fail) to account for ourselves using CT.

Outline

Wiring diagrams

We can imagine making new systems from old by wiring them together:



Joyal-Street invented string diagrams for monoidal cats:

- They're up-to-homotopy 1-d submanifolds of $\mathbb{R}^2$ with some properties.
- With Verity, invented traced monoidal cats: including loops as above.

Wiring diagrams add the outer box: noticing operadic structure.

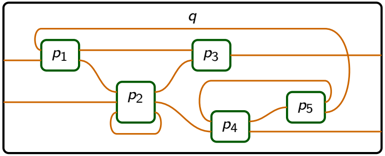
- You can nest WDs inside of each box of a WD.
- The operad for traced categories is End(1-$\mathbf{Cob}, 0, +$), i.e...
- ...up to $\simeq$, a traced cat'y is a functor 1-$\mathbf{Cob}_{/O} \to \mathbf{Set}$ for some set O.
- Boxes are signed sets, WDs are cobordisms, a syntax for composition.

Algebras for wiring diagram operads

Let $\mathscr{W}$ be an operad; call its objects *boxes*, morphisms *wiring diagrams*.

- A functor $F\colon \mathscr{W} \to \mathbf{Set}$ assigns a set of *fillers* to each box and...
- ... an op'n taking inner fillers to outer filler for each $\varphi\colon b_1, \ldots, b_k \to b'$.
- Again, a functor $F\colon 1\text{-}\mathbf{Cob}_{/O} \to \mathbf{Set}$ is a traced category.

$\varphi =$



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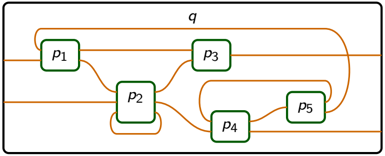
$\varphi =$  $: p_1, \ldots, p_5 \to q$

Almost example: a box can be filled by any system of input-output ODEs.

- Your (or a cell's) body as interface: the signals it can send and receive.
- Say we send elements of $\mathbb{R}^m$ and receive elements of $\mathbb{R}^n$.
- $F\colon 1\text{-}\mathbf{Cob}_{/O} \xrightarrow{?} \mathbf{Set}$ by $F(m, n) := \{(\mathbb{R}^m, \mathbb{R}^n)\text{-ODE systems}\}$,

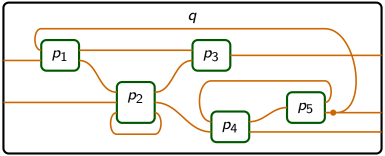
$$\dot{x} := f(x, a) \quad b := g(x), \quad \text{where} \quad \ell : \mathbb{N}, x : \mathbb{R}^\ell, a : \mathbb{R}^m, b : \mathbb{R}^n$$

- Fill each inner box with an io-ODE system (f_i, g_i); then...
- ...$F(\varphi)((f_1, g_1), \ldots, (f_k, g_k))$ fills$^?$ the outer box with an ODE system.

Algebras for exotic wiring diagram operads

But the above doesn't quite work: ODE systems don't form a traced cat'y.

- Reason: no identity. The io-ODE $(\dot{x} := a - x, b := x)$ $a \boxed{x \circlearrowleft} b$?
- Traced cats without identity? Weird! Axioms fall apart? What to do?
- Answer: just change your wiring diagram operad! Allow splits too?



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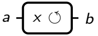
- Reason: no identity. The io-ODE $(\dot{x} := a - x, b := x)$  ?
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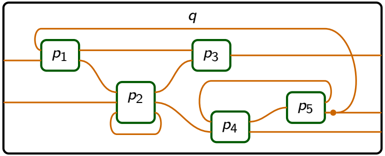

The operadic view: structured cats are semantics for wiring as syntax.

- Forget or add structure: $\mathscr{W} \to \mathscr{W}'$. But we can do much more.
- Using intuition from $\mathscr{W}$-**Alg**, we can make wiring way more fancy.

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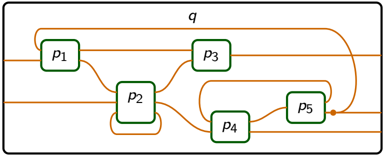

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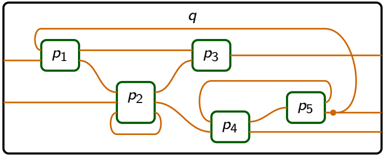

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What's the math for WDs that change based on what flows through them?

The category of polynomial functors

Poly is a very elegant and expressive category for modeling such systems.

- The cat'y **Poly** is the free completely distributive category on a point.
- Objects are connected-limit preserving functors $p\colon \mathbf{Set} \to \mathbf{Set}$.
- Equivalently, arbitrary coproducts of representables $p = \sum_{i:I} y^{A_i}$.
- Morphisms $\varphi\colon p \to q$ are nat'l transf'ns. By Yoneda and coprod UP,

$$\mathbf{Poly}\Big(\sum_{i:I} y^{A_i}, \sum_{j:J} y^{B_j}\Big) \cong \prod_{i:I}\sum_{j:J}\prod_{b:B_j}\sum_{a:A_i} 1.$$

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Poly has a lot of structure. Some highlights:

- **Poly** has all limits and colimits, at least three or. factorization systems.
- It is cartesian closed. It has infinitely-many monoidal closed structures.
- Indeed each monoidal $\cdot$ on **Set** lifts to distrib've monoidal $\odot$ on **Poly**.
- Substitution (functor comp'n) $\triangleleft$ is monoidal, has left Kan (coclosure).
- $\triangleleft$ and $\otimes$ are normal duoidal: $(p_1 \triangleleft p_2) \otimes (q_1 \triangleleft q_2) \to (p_1 \otimes q_1) \triangleleft (p_2 \otimes q_2)$.

Polynomial coalgebras

For $F\colon \mathbf{Set} \to \mathbf{Set}$, an F-coalgebra is a set S and a map $S \to F(S)$.

- For any polynomial p, the cat'y of p-coalgebras forms a topos.
- Let's take $p = By^A$. Then $S \to BS^A$ is $r\colon S \to B$ and $u\colon S \times A \to S$.
  - This is called a *Moore machine* with inputs A and outputs B.
  - A *state* $s{:}S$ *reads out* $r(s){:}B$ and *updates to* $u(s, a){:}S$ given $a{:}A$.

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We think of a map $\varphi\colon p_1 \otimes \cdots \otimes p_K \to p'$ as an *interaction pattern*.

- It transforms an output of the p_k's to: an output of p', toge'r with...
- ...a function from inputs of p' to inputs of each p_k.
- $\varphi : \prod_{b_1:B_1} \cdots \prod_{b_K:B_K} \sum_{b':B'} \prod_{a':A'} A_1 \times \cdots \times A_K$

Here's the formula for Day-$\times$ internal hom $[p_1 \otimes \cdots \otimes p_K, p']$.

$$\left[p_1 \otimes \cdots \otimes p_K, p'\right] \cong \sum_{\varphi\colon\, p_1 \otimes \cdots \otimes p_K \to p'} y^{B_1 \times \cdots \times B_K \times A}$$

So what is a $[p_1 \otimes \cdots \otimes p_k, q]$-coalgebra $S \to [p_1 \otimes \cdots \otimes p_k, q](S)$?

- It's a state machine with states S, which outputs interaction patterns...
- ...and updates based on what flows: outputs of p's and input of q.

Rewiring diagrams and updating interfaces

So we can greatly increase our "wiring syntax" by allowing rewiring.

- Define $\mathbb{O}\mathbf{rg}$ to be the monoidal bicat'y with objects $\mathrm{Ob}\,\mathbb{O}\mathbf{rg} = \mathrm{Ob}\,\mathbf{Poly}$
- and $\mathbb{O}\mathbf{rg}(p, q) := [p, q]\text{-}\mathbf{coAlg}$: *rewiring diagrams*.
- Def: a *dynamic* X (cat'y, op'd, etc) is an X enriched in $\mathbb{O}\mathbf{rg}$.

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Example: Training neural networks is a dynamic prop.

- There's a strong mon'l functor $t\colon \mathbf{Mfd} \to \mathbf{Poly}$ by $M \mapsto \sum_{m:M} y^{T^*_m M}$.
- It sends Lie gps to Hopf monoids in $(\mathbf{Poly}, \otimes)$, e.g. $t(\mathbb{R})^{\otimes N} \cong t(\mathbb{R}^N)$.
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More generally, we can change not only the wiring but also the interfaces.

- A body can lose a leg or gain a prosthetic, based on what's sent to it.
- This can model cells that change their receptors through time.
- No time today, but see arXiv: 2602.17917 if interested.

Outline

Summary

Life invents coordination structures at every scale.

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Maybe inventing accounting systems *is* what accounts for our inventiveness.

Thank you for attending! Comments and questions welcome...